Data-aided and blind synchronization

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Outline

1. **Preamble-based joint CFO and symbol timing estimation**
   - LS estimator
   - ML estimator
   - Performance evaluation

2. **Blind synchronization**

3. **Preamble-based synchronization in uplink**

4. **Conclusions**

FBMC signal model

Received signal in the presence of synchronization errors

$$r(t) = e^{j2\pi\left(\frac{\varepsilon}{T}t+\phi\right)}|c|\,s(t-\tau)+n(t)$$

where

$$s(t) = \sum_{k\in \mathrm{M}_u}\sum_{n=-\infty}^{+\infty} d_{k,n}\beta_{k,n}\theta_{k,n}\,p\left(t-n\frac{T}{2}\right)e^{j\frac{2\pi}{T}kt}$$

and

ε = carrier frequency offset (CFO) normalized to the subcarrier spacing

ϕ = carrier phase offset

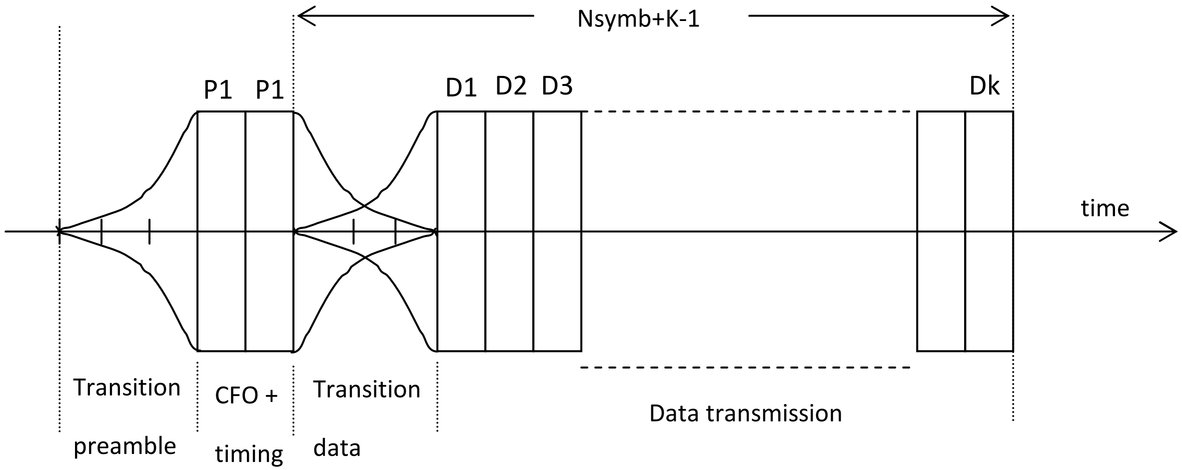
τ = timing offset

$|c|$ = attenuation

$n(t)$ = white gaussian noise

Preamble-based joint CFO and timing estimation – LS approach

Structure of the burst

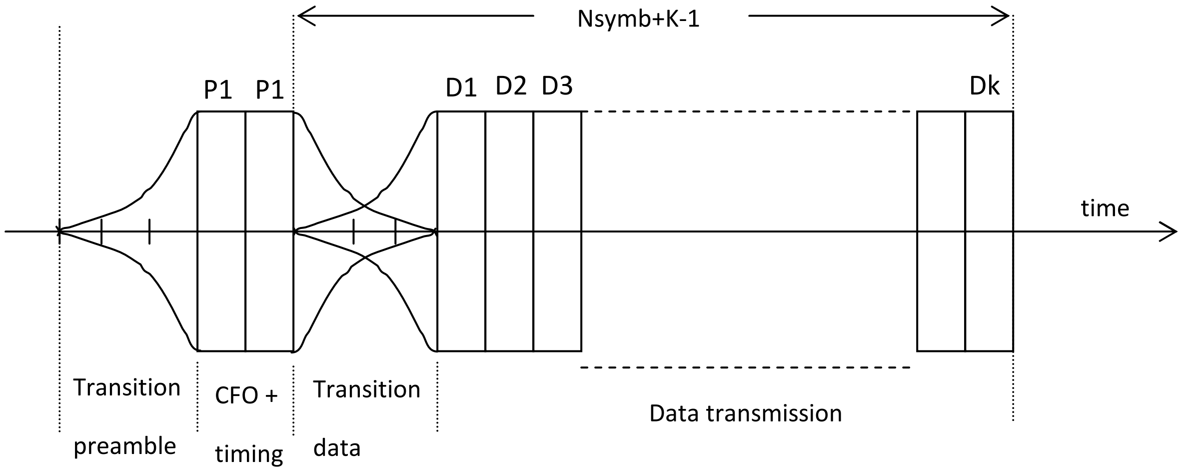


The joint CFO and symbol timing (ST) estimator based on the least squares (LS) approach exploits the transmission of a training sequence made up of two identical FBMC symbols.

$$\begin{cases} \hat{\tau}_{LS} = \arg\max_{\tilde{\tau}} \left\{ 2\left|R(\tilde{\tau})\right| - Q(\tilde{\tau}) \right\} \\ \hat{\varepsilon}_{LS}(\hat{\tau}_{LS}) = \frac{1}{2\pi} \angle \left\{ R(\hat{\tau}_{LS}) \right\} \end{cases} \text{where} \begin{cases} R(\tilde{\tau}) = \sum_{m=(K-1)M}^{KM-1} r^*[m+\tilde{\tau}/T_s]\,r[m+M+\tilde{\tau}/T_s] \\ Q(\tilde{\tau}) = \sum_{m=(K-1)M}^{KM-1} |r[m+\tilde{\tau}/T_s]|^2 + \sum_{m=(K-1)M}^{KM-1} |r[m+M+\tilde{\tau}/T_s]|^2 \end{cases}$$

Preamble-based joint CFO and timing estimation – LS approach

Structure of the burst



Properties of the derived estimator

- It does not require the knowledge of the SNR.
- It provides a closed-form solution for the CFO estimate and can assure unambiguous CFO estimates if $|\varepsilon| \leq 1/2$.
- It exploits the transmission of a short preamble composed by two identical FBMC symbols, or one FBMC symbol with two identical parts.

Preamble-based joint CFO and timing estimation – ML approach

The proposed joint ML phase offset, CFO and symbol timing estimator exploits the transmission of a known training sequence of N_{TR} symbols.

$$\begin{cases} \hat{\phi}_{ML}(\tilde{\varepsilon},\tilde{\tau}) = \dfrac{1}{2\pi}\angle\left\{ \displaystyle\sum_{m=0}^{\eta M-1} r[m]\tilde{z}[m-\tilde{\tau}/T_s]^* e^{-j\frac{2\pi\tilde{\varepsilon}}{M}m} \right\} \\ (\hat{\tau}_{ML},\hat{\varepsilon}_{ML}) = \underset{(\tilde{\tau},\tilde{\varepsilon})}{\arg\max}\left[\left| \displaystyle\sum_{m=0}^{\eta M-1} r[m]\tilde{z}[m-\tilde{\tau}/T_s]^* e^{-j\frac{2\pi\tilde{\varepsilon}}{M}m} \right| \right] \end{cases}$$

where $z[m] = \sum_{k\in M_u} \sum_{n=0}^{2N_{TR}-1} d_{k,n}\theta_{k,n}\beta_{k,n}\, p[m-nM/2]\, e^{j\frac{2\pi}{M}km}$

The joint ML estimator evaluates, for each trial value of the ST $\tilde{\tau}$ and of the CFO $\tilde{\varepsilon}$, the response of the matched filter $p[\cdot]$ to the CFO compensated and down converted signal. In the down-conversion, the frequencies of all active subcarriers are considered.

Preamble-based joint CFO and timing estimation – ML approach

Assumptions

- CFO sufficiently small
- Training sequence of lenght N_{TR}=1 symbol

The joint ML estimator can be approximated (AML) as

$$\begin{cases} \hat{\tau}_{AML} = \arg\max_{\tilde{\tau}} \left\{ \left| A(\tilde{\tau}) \right| + \left| B(\tilde{\tau}) \right| \right\} \\ \hat{\varepsilon}_{AML} = \frac{1}{\pi} \angle \left\{ A^*\left(\hat{\tau}_{AML}\right) B\left(\hat{\tau}_{AML}\right) \right\} \end{cases} \text{where} \begin{cases} A(\tilde{\tau}) = \sum_{k \in M_u} d_{k,0} \theta_{k,0}{}^* \beta_{k,0}{}^* w_0^{(k)}\left(0, \tilde{\tau}\right) \\ B\left(\tilde{\tau}\right) = \sum_{k \in M_u} d_{k,1} \theta_{k,1}{}^* \beta_{k,1}{}^* w_1^{(k)}\left(0, \tilde{\tau}\right) \\ w_n^{(k)}\left(0, \tilde{\tau}\right) = \sum_{m=0}^{\eta M - 1} r[m] p[m - nM/2 - \tilde{\tau}/T_s] e^{j\frac{2\pi}{T}k(\tilde{\tau} - m)} \end{cases}$$

Properties of the derived AML estimator

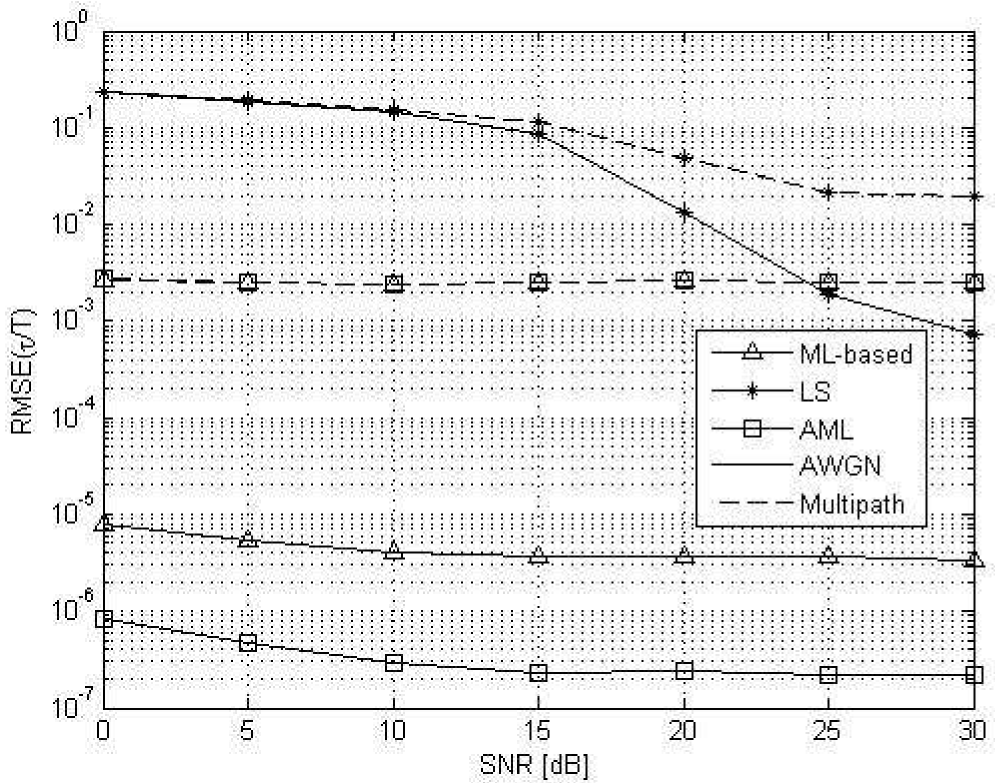
- It does not require the knowledge of the SNR
- It can assure accurate estimates in a quite wide range and also when the preamble is composed of only one FBMC symbol
- It can be exploited in an up-link scenario

Performance Evaluation

Simulation Parameters:

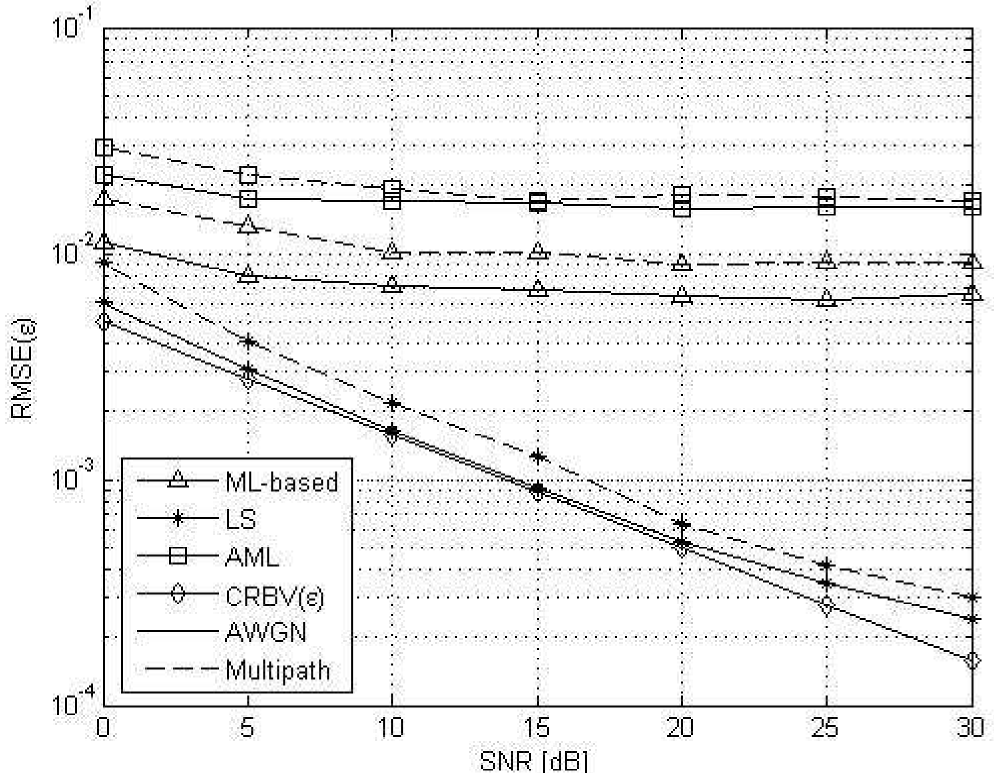
- The values of the normalized CFO and of the timing offset are uniformly distributed in [-0.25, 0.25) and T_s{-M/2,…,M/2-1}, respectively. In particular, the ST is supposed to be an integer multiple of the sampling period T_s.
- The size of the set of subcarriers is M=1024.
- The joint LS CFO and ST estimator exploits a preamble made up of two identical FBMC symbols.
- The considered joint AML and ML-based estimators exploit a preamble made up of only one FBMC symbol.
- The prototype filter has an overlap parameter K=4.
- Numerical results have been obtained by considering two different scenarios using 4-QAM constellation: AWGN channel and ITU-R multipath fading Vehicular-A channel. Moreover, the channel is fixed in each run but it is independent from one run to another.

Preamble-Based Estimators – Symbol Timing Estimation Performance



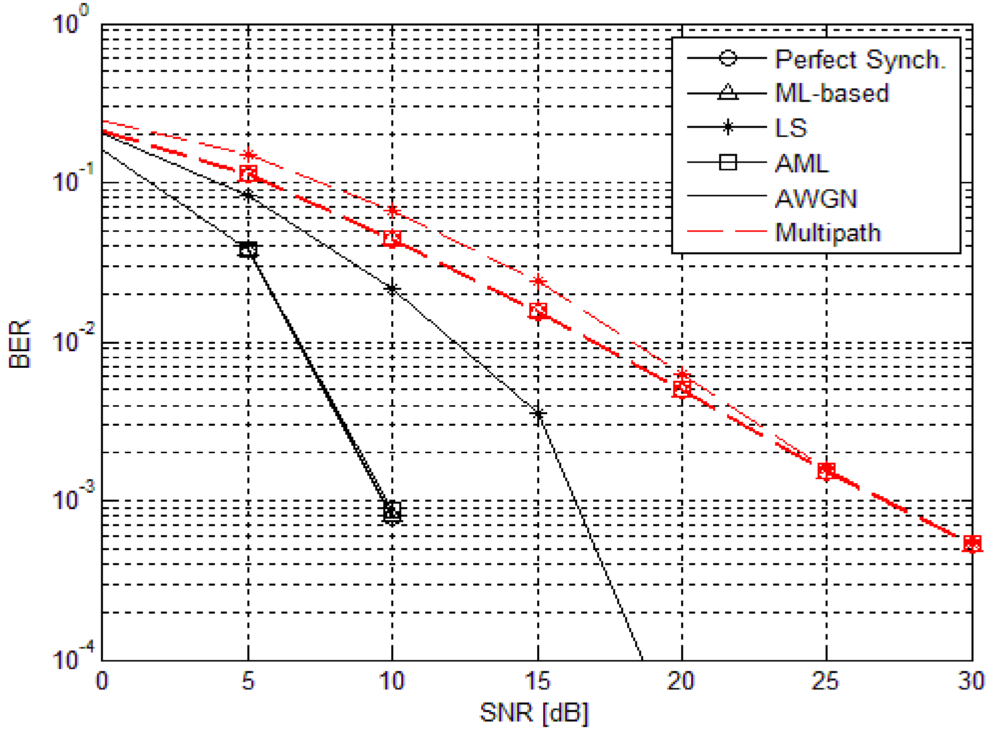
In multipath channel the AML and ML-based ST estimators exhibit a significant performance degradation with respect to the AWGN case.

Preamble-Based Estimators – CFO Estimation Performance



The LS CFO estimator provides in AWGN channel a performance very close to the bound, it assures a significant gain with respect to the ML-based and the AML techniques and, moreover, it does not present a floor in multipath channel.

Preamble-Based Estimators – BER Performance



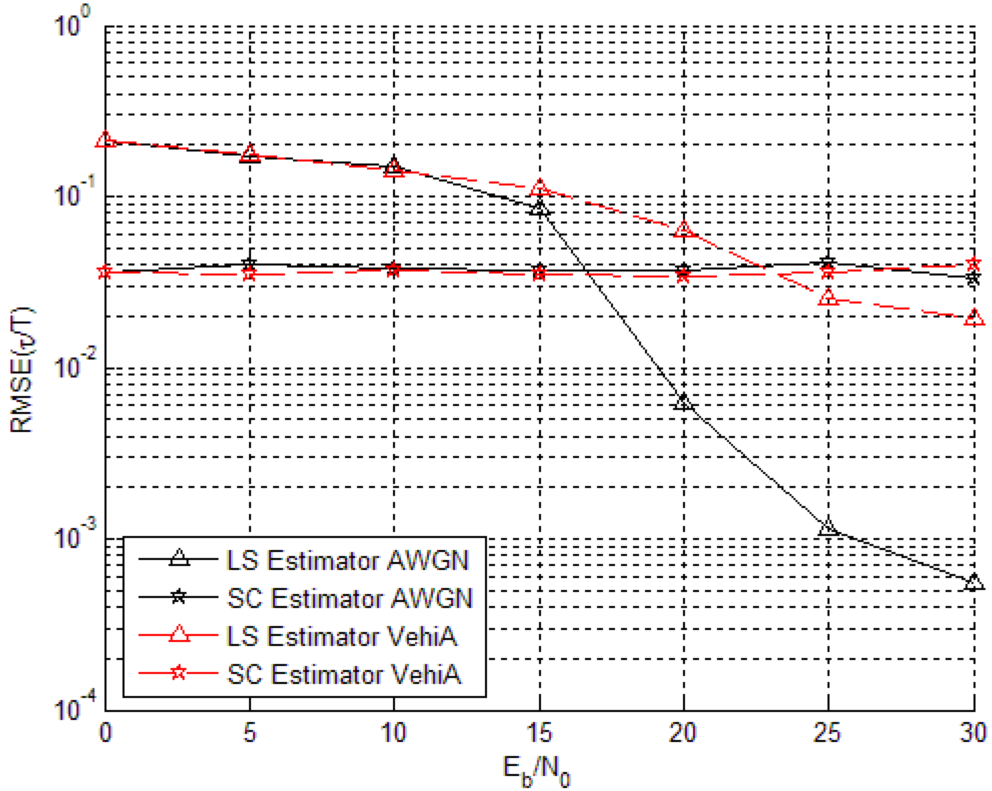
In AWGN the ML-based estimators assure the best performance in terms of BER. In multipath channel the ML-based, the AML and the LS estimators provide a BER very close to that obtained in the case of perfect synchronization.

Comparisons with OFDM Systems

Simulation Conditions:

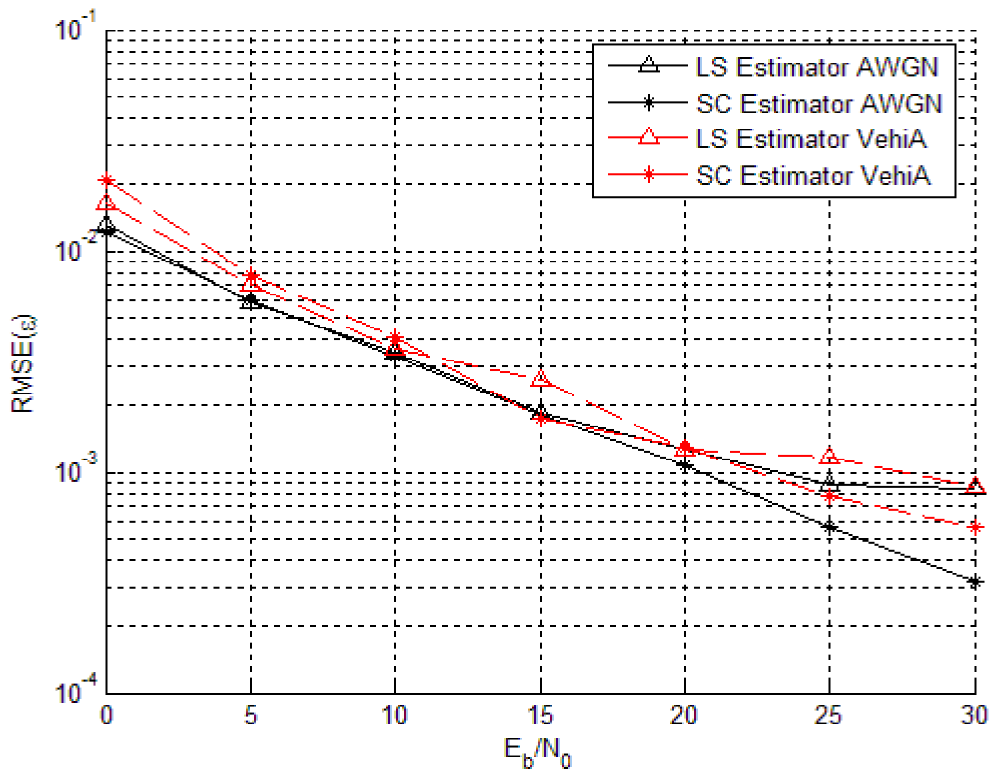
- The values of the normalized CFO and of the timing offset are uniformly distributed in [-0.5, 0.5) and $T_s\{-M/2,\ldots,M/2-1\}$, respectively.
- The size of the set of subcarriers for the considered OFDM system is M=1024 while the cyclic prefix length is CP=M/16.
- The OFDM training symbol has two identical parts each of length M/2=512.
- The FBMC training symbol with two identical parts has been obtained by applying at the input of the IFFT in the transmitter *K* identical preamble symbols, with pseudo-noise data only on even subcarriers.
- The multipath channel has been modelled using the Vehicular-A channel model of ITU-R.

Comparisons with OFDM Systems – Symbol Timing Estimation



The results show that the LS estimator outperforms the Schmidl and Cox (SC) estimator when E_b/N_0 is sufficiently high.

Comparisons with OFDM Systems – CFO Estimation



Both in AWGN and multipath channel the performance of the LS and SC CFO estimators is very similar.

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Signal model

Received signal in the presence of synchronization errors

$$r[k] = e^{j2\pi\left(\frac{\varepsilon}{M}k+\phi\right)} |c| s[k - \tau / T_s] + n[k]$$

where

$$s[k] = \sum_{l \in \mathrm{M}_u} \sum_{n=-\infty}^{+\infty} d_{l,n} \beta_{l,n} \theta_{l,n} p\left(k - n\frac{M}{2}\right) e^{j\frac{2\pi}{M}lk}$$

and

$\varepsilon =$ CFO normalized to the subcarrier spacing

$\phi =$ carrier phase offset

$\tau =$ symbol timing offset

$|c| =$ attenuation

$n[k] =$ white Gaussian noise

Blind synchronization

Cyclostationarity properties of the OFDM/OQAM signal

Assumptions

- The data symbols, belonging to a PAM constellation, are i.i.d. random variables with zero mean and unit variance.
- The real-valued and unit energy pulse-shaping filter p[k] is band-limited within [-1/M,1/M].

Under these assumptions, it results that:

- The **Unconjugate correlation function** of the transmitted signal s[k], at time k and lag m is given by

$$R_s(k,m) = E\left[s^*[k]s[k+m]\right] = \frac{2}{M}\sum_{l\in \mathrm{M}_u} e^{j\frac{2\pi}{M}lm} \int_{-1/2}^{1/2} \left|P(\nu)\right|^2 e^{-j2\pi\nu m} d\nu, \qquad P(\nu) = \sum_{k=-\infty}^{+\infty} p[k] e^{-j2\pi k\nu}$$

- The **Conjugate correlation function** (or relation function) of the transmitted signal s[k], at time k and lag m is given by

$$C_s(k,m) = E\left[s[k]s[k+m]\right] = \sum_{l\in \mathrm{M}_u} e^{j\frac{2\pi}{M}l(2k+m)} \sum_{n=-\infty}^{+\infty} \left(\theta_{n,l}\beta_{n,l}\right)^2 p\left[k-\frac{nM}{2}\right] p\left[k+m-\frac{nM}{2}\right]$$

Blind ML CFO Synchronization

Blind ML CFO Synchronization

Assumptions:

- Low SNR values.
- Ideal symbol timing recovery.

$$\begin{cases} \hat{\phi}_{ML} = \dfrac{1}{4\pi} \angle \left\{ \displaystyle\sum_{k_1=0}^{\eta M-1} \sum_{k_2=0}^{\eta M-1} \left(r[k_1] r[k_2] \right)^* \mathrm{E}\left[\tilde{s}^{\tilde{\varepsilon}}[k_1] \tilde{s}^{\tilde{\varepsilon}}[k_2] \right] \right\} \\ \hat{\varepsilon}_{MLLS} = \arg\max_{\tilde{\varepsilon}} \left\{ \left| \displaystyle\sum_{k_1=0}^{\eta M-1} \sum_{k_2=0}^{\eta M-1} \left(r[k_1] r[k_2] \right)^* \mathrm{E}\left[\tilde{s}^{\tilde{\varepsilon}}[k_1] \tilde{s}^{\tilde{\varepsilon}}[k_2] \right] \right| + \sum_{k_1=0}^{\eta M-1} \sum_{k_2=0}^{\eta M-1} r[k_1] r[k_2]^* \mathrm{E}\left[\tilde{s}^{\tilde{\varepsilon}}[k_1] \tilde{s}^{\tilde{\varepsilon}}[k_2]^* \right] \right\} \end{cases}$$

- The derived blind MLLS CFO estimator depends on both the conjugate and unconjugate correlation functions of the transmitted signal.
- The blind CFO estimator exploiting only the first term of $\hat{\varepsilon}_{MLLS}$ depends only on the conjugate correlation function (CMLLS estimator).
- The blind CFO estimator exploiting only the second term of $\hat{\varepsilon}_{MLLS}$ depends only on the unconjugate correlation function (UMLLS estimator) (the system should not be fully loaded).

Blind Synchronization

Simplified CFO Synchronization

$$\hat{\varepsilon}_{LS} = \frac{1}{2\pi}\angle\left\{\sum_{m=L_1}^{L_2-1}\hat{R}(m)\hat{R}(N-m)\right\} \quad \text{where} \quad \begin{cases} \hat{R}(m) = \dfrac{1}{M-m}\displaystyle\sum_{k=0}^{M-m-1} r^*[k]r[k+m] \\ \hat{R}(M-m) = \dfrac{1}{m}\displaystyle\sum_{k=0}^{m-1} r^*[k]r[k+M-m] \end{cases}$$

L1 and L2 are design parameters selected so that the conditions N-m>>1 and m>>1 are satisfied.

The proposed LS CFO estimator:

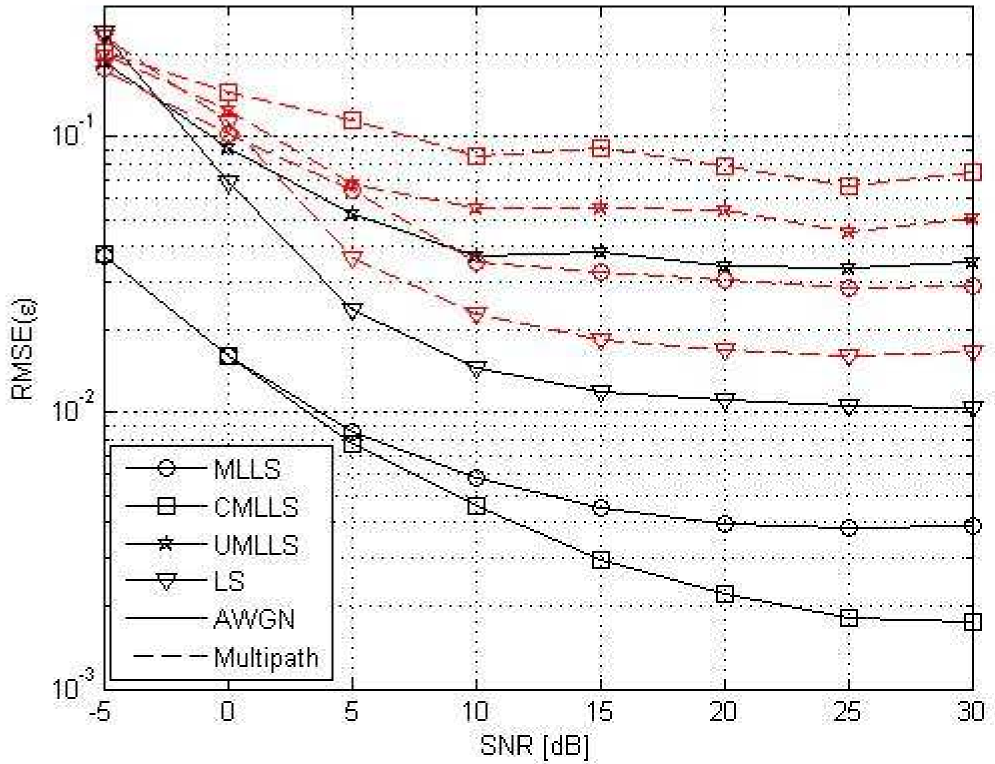
- is based on the least squares approach,
- its acquisition range is $|\varepsilon| \leq 1/2$,
- does not require the knowledge of the SNR.

Performance Evaluation of CFO estimators

Simulation Conditions:

- The value of the CFO is uniformly distributed in [-0.5, 0.5).
- The size of the set of subcarriers is M=1024 and the number of virtual subcarriers is M_v=128.
- The prototype filter has an overlap parameter *K*=4.
- Numerical results have been obtained by considering two different scenarios: AWGN channel and multipath channel modelled using the Vehicular-A channel model of ITU-R. Moreover, the channel is fixed in each run but it is independent from one run to another.

Performance Evaluation of CFO estimators



The results (with η=1) show that in AWGN channel the MLLS and CMLLS estimators provide the lowest RMSE while, in the considered multipath channel, the LS estimator assures the most accurate estimates.

Blind Synchronization

Blind ML Symbol Timing Synchronization

Assumptions:

- Low SNR values.
- Ideal CFO recovery.

$$\hat{\tau}_{MLLS} = \arg\max_{\tilde{\tau}} \left[\left| \sum_{k_1=0}^{\eta M-1} \sum_{k_2=0}^{\eta M-1} r[k_1] r[k_2] \mathrm{E}\left[\tilde{s}^{\tilde{\tau}}[k_1]^* \tilde{s}^{\tilde{\tau}}[k_2]^* \right] \right| \right]$$

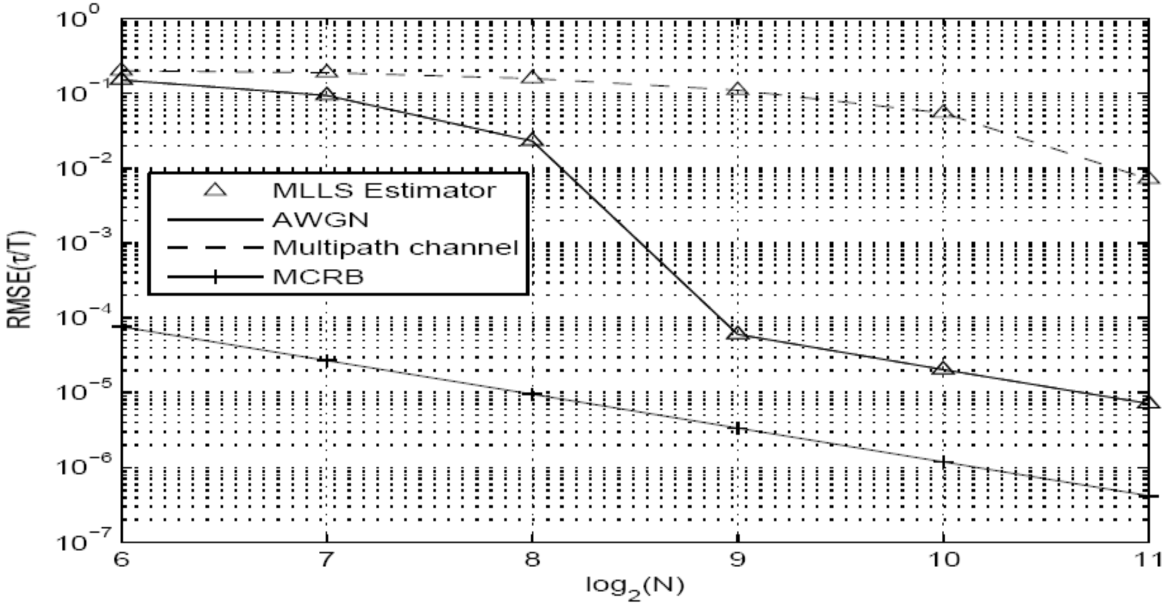
The derived blind symbol timing estimator is based on the ML approach. It depends only on the conjugate correlation function of the transmitted FBMC signal and requires an one-dimensional maximization procedure.

Performance Evaluation of the blind ST estimator

Simulation Conditions:

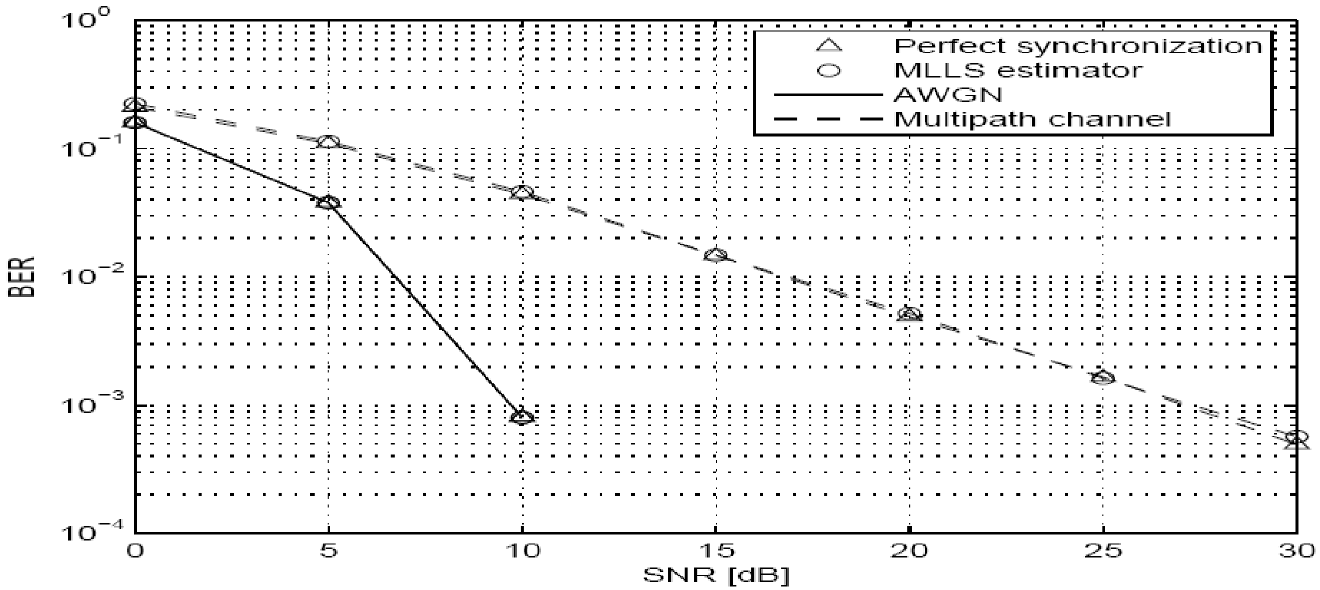
- The value of the timing offset is uniformly distributed in $T_s\{M/4,\ldots,M/4-1\}$.
- The size of the set of subcarriers is M=1024.
- The prototype filter has an overlap parameter K=4.
- Numerical results have been obtained by considering two different scenarios: AWGN channel and multipath channel modelled using the Vehicular-A channel model of ITU-R. Moreover, the channel is fixed in each run but it is independent from one run to another.

Performance Evaluation of Symbol Timing estimator



The results (with 20 dB SNR and η=1) show that in AWGN channel, the proposed estimator provides accurate estimates and, moreover, the asymptotic (M>>1) slope is similar to that predicted by the MCRB. In the considered multipath channel, the ML estimator exhibits a performance degradation due to the mismatch with respect to the considered model. However, the estimates result to be quite accurate for a sufficiently large number of subcarriers. When a large number of subcarriers is used and only one symbol FBMC is exploited the accuracy of the estimates is not affected by the CFO.

Performance Evaluation of Symbol Timing estimator



The adoption of the proposed algorithm assures a negligible performance degradation with respect to the case of one-tap channel equalization with perfect channel knowledge.

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Multiple access signal model

Received signal in the presence of synchronization errors

$$r[m] = \sum_{i=1}^{U} e^{j2\pi\left(\frac{\varepsilon_i}{M}k+\phi_i\right)} |c_i| s_i[m - \tau_i / T_s] + n[m]$$

where

$$s_i[m] = \sum_{l \in \mathrm{M}_u^i} \sum_{n=-\infty}^{+\infty} d_{l,n} \beta_{l,n} \theta_{l,n} p\left(m - n\frac{M}{2}\right) e^{j\frac{2\pi}{M}lm}$$

and

U = number of users

ε_i = CFO normalized to the subcarrier spacing of the i-th user

ϕ_i = carrier phase offset of the i-th user

τ_i = timing offset of the i-th user

$|c_i|$ = attenuation of the i-th user

$n[m]$ = white Gaussian noise

Synchronization and Initialization in Uplink

Joint CFO and symbol timing estimator

Under the assumption that the CFO of the i-th user is sufficiently small and into the case of a training sequence of length N_{TR} =1 FBMC symbol, the ML approach leads to U different joint estimators estimator

$$\begin{cases} \hat{\tau}_i = \arg\max_{\tilde{\tau}_i} \left\{ \left|A(\tilde{\tau})\right| + \left|B(\tilde{\tau})\right| \right\} \; i=1,\dots,U \\ \hat{\varepsilon}_i(\hat{\tau}_i) = \frac{1}{\pi} \angle \left\{ A^*(\hat{\tau}_i) B(\hat{\tau}_i) \right\} \; i=1,\dots,U \end{cases} \text{ where } \begin{cases} A(\tilde{\tau}_i) = \sum_{k \in M^i_u} d_{k,0} \theta_{k,0}{}^* \beta_{k,0}{}^* w_0^{(k)}(0,\tilde{\tau}_i) \\ B(\tilde{\tau}_i) = \sum_{k \in M^i_u} d_{k,1} \theta_{k,1}{}^* \beta_{k,1}{}^* w_1^{(k)}(0,\tilde{\tau}_i) \\ w_n^{(k)}(0,\tilde{\tau}_i) = \sum_{m=0}^{\eta M-1} r[m] p[m - nM/2 - \tilde{\tau}_i / T_s] e^{j\frac{2\pi}{T}k(\tilde{\tau}_i - m)} \end{cases}$$

Properties of the derived estimator

- It exploits a short known preamble embedded in the burst received from each of U users.
- The CFO estimate is in closed form, while the symbol timing estimate requires an one-dimensional maximization procedure.

Performance Evaluation of the proposed joint estimator

Simulation Conditions

- The values of the CFO and of the timing offset of each user are uniformly distributed in [-0.5, 0.5) and T_s{-M/2,M/2}, respectively.

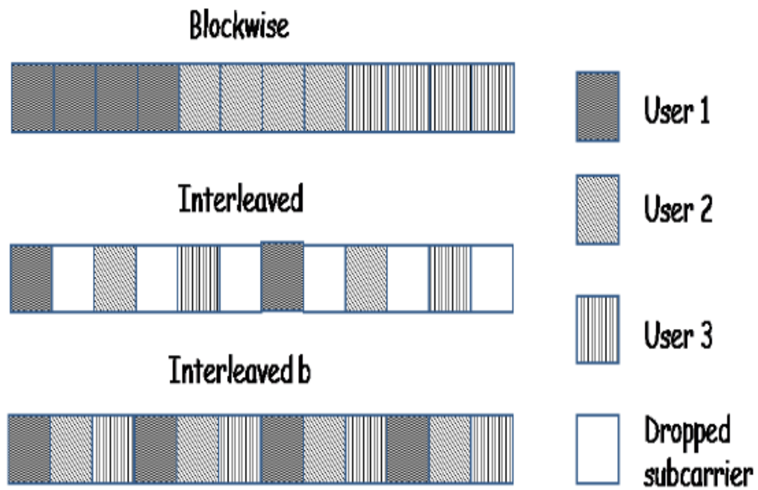
- The size of the set of subcarriers is M=1024 while the number of users is U=4 and M_u^i =256, i = 1,…,4.

- The preamble is made up of N_{TR}= 1 FBMC symbol.

- Numerical results have been obtained by considering two different scenarios: AWGN channel and multipath channel modelled using the Vehicular-A channel model of ITU-R. Moreover, the channel is fixed in each run but it is independent from one run to another.

Performance Evaluation of the proposed joint estimator

Allocation schemes

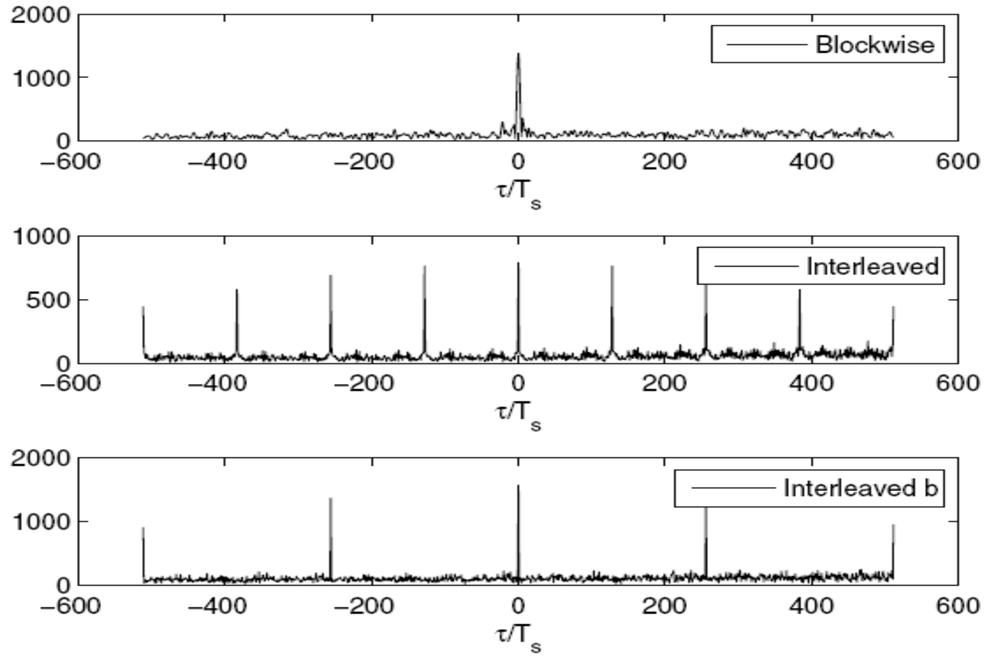


Blockwise	adjacent subcarriers are allocated to the same user
Interleaved	adjacent subcarriers are allocated to different users and one subcarrier is dropped between two users
Interleaved b	adjacent subcarriers are allocated to different users

Performance Evaluation of the proposed joint estimator

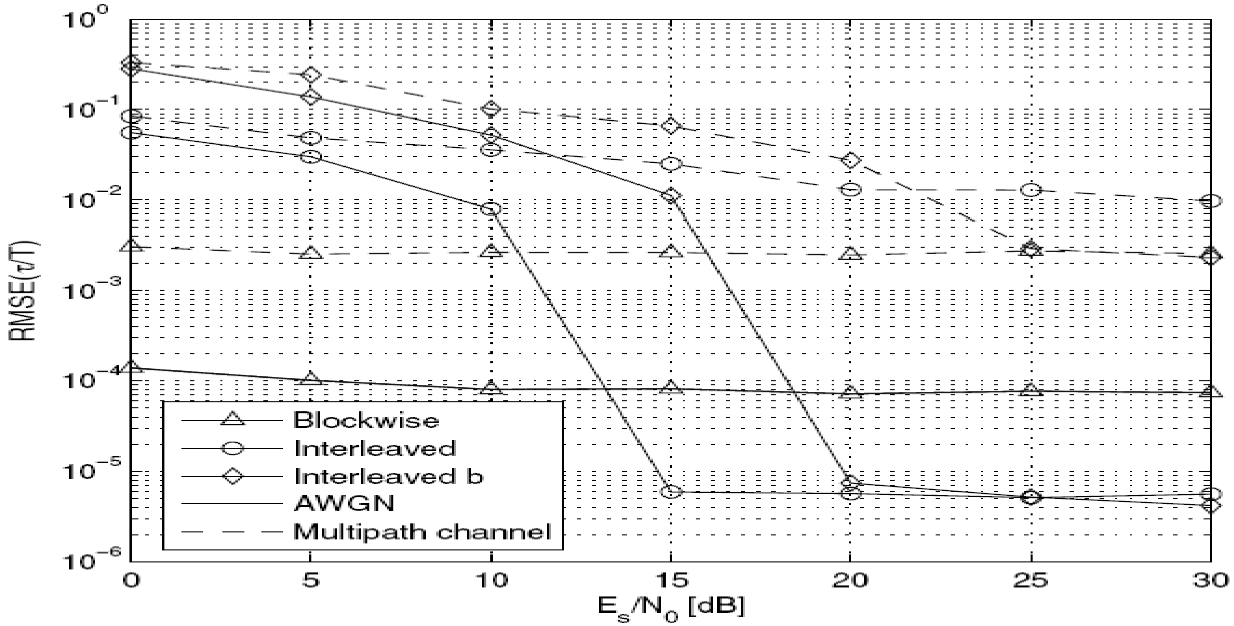
Cost function

U = 4 users
N = 1024 subcarriers
L = 2 training simbols
E_s/N_0 = 13dB for each user



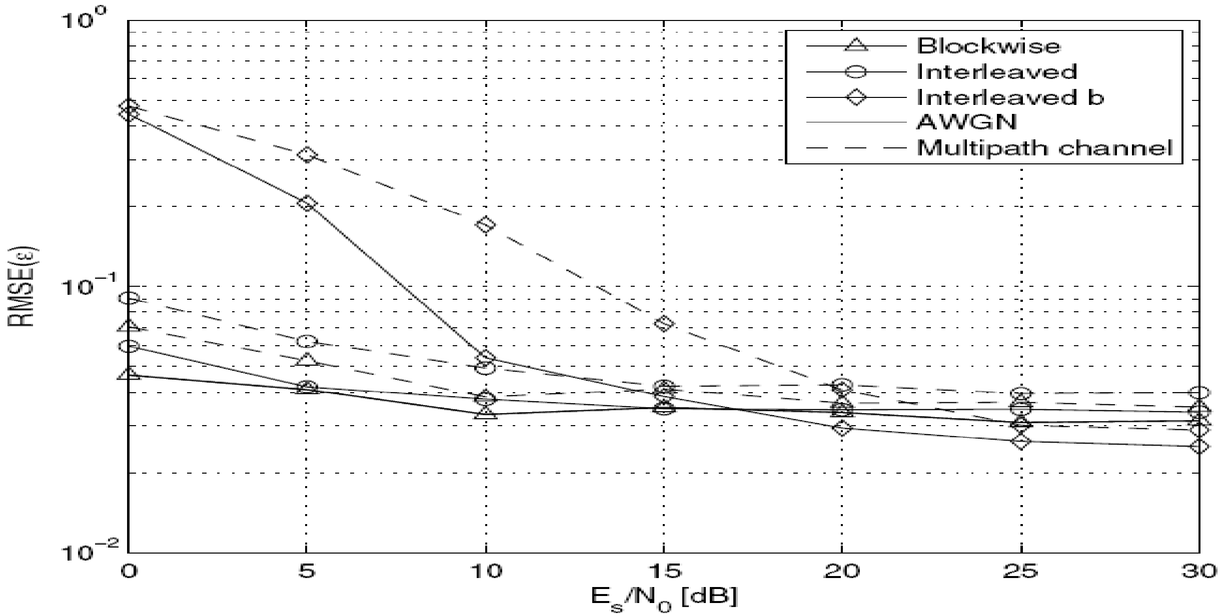
The behaviour of the cost function depends on the adopted allocation scheme. In the case of interleaved and interleaved b schemes, the cost function presents local maxima that can interfere with the absolute maximum especially for low SNR values or in multipath channels, while in the case of blockwise allocation, the considered ML cost function exhibits only one sharp peak at the actual value of the timing offset τ_1=0.

Performance Evaluation of the proposed joint estimator



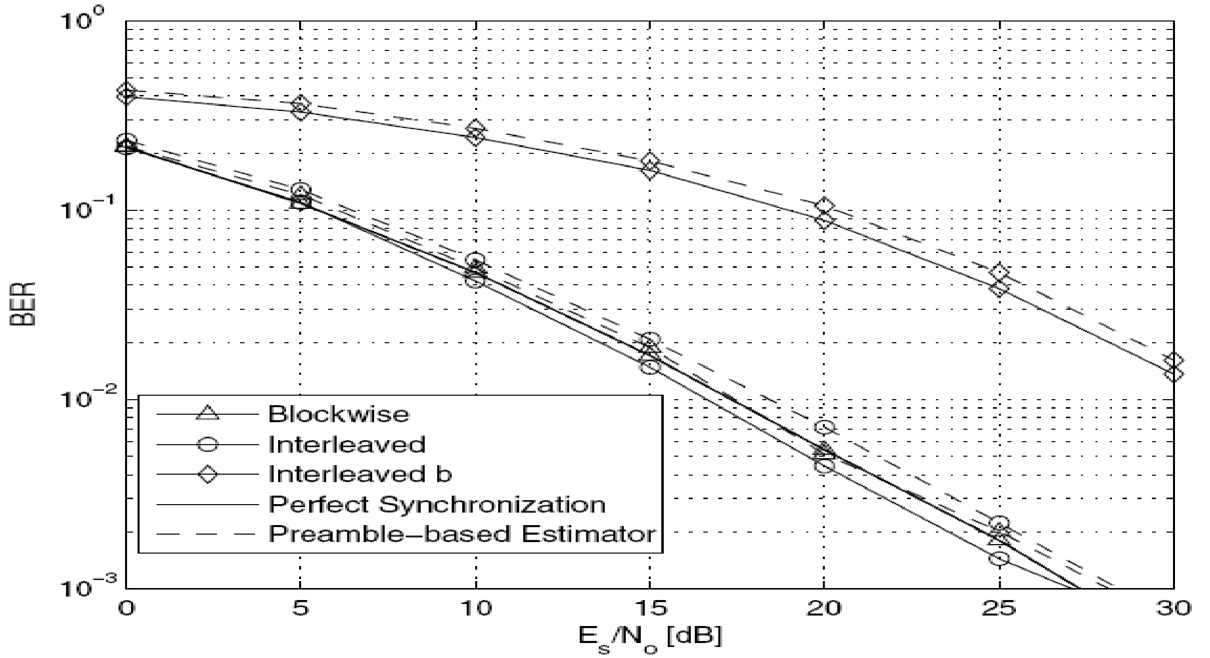
In AWGN channel and, for sufficiently high Es/No values, the interleaved and interleaved b schemes assure better estimates than blockwise scheme. However, the presence of local maxima in the cost functions for the interleaved and interleaved b schemes leads to a severe performance degradation as Es/No decreases. The effect of the local maxima is also evident from the performance of the interleaved b scheme in multipath channel.

Performance Evaluation of the proposed joint estimator



Both in AWGN and multipath channel, for sufficiently high E_s/N_0 values, the lowest RMSE is obtained when the interleaved b scheme is adopted.

Performance Evaluation of the proposed joint estimator



For all the considered allocation schemes, a cointained performance loss is observed with respect to the case of one-tap equalization with perfect channel knowledge and perfect synchronization.

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Conclusions

Preamble-based Estimation

- A joint CFO and symbol timing estimator based on the LS approach exploiting the transmission of a training sequence made up of two identical FBMC symbols has been derived.
- The ML joint CFO and ST estimator for AWGN channel and its approximated (AML) version for small CFO values have been obtained.
- The LS estimator can provide in multipath channel a BER very close to that obtained in the case of perfect synchronization, when the number of subcarriers is sufficiently large.
- The AML estimator can assure a negligible performance loss with respect to the perfectly synchronized system.

Conclusions

Blind Synchronization

- The ML CFO and ST estimators for low SNR values have been derived.
- A symplified CFO estimator based on the LS approach has been proposed.
- The LS CFO estimator assures satisfactory estimates in multipath channel.
- The ML symbol timing estimator assures a contained performance degradation with respect to the case of one-tap channel equalization with perfect channel knowledge.
- If the number of subcarriers is sufficiently large and only one multicarrier symbol is exploited, the performance of the blind symbol timing estimator is not affected by the CFO.

Conclusions

Preamble-based synchronization in uplink

- The ML joint estimator for CFO and ST of each of U users has been proposed.
- Three kinds of subcarrier allocation schemes (blockwise, interleaved and interleaved b) have been considered.
- In multipath channel, the proposed ML ST estimator provides the best estimate in the case of blockwise scheme.
- For sufficiently high E_s/N_0 values, the derived ML CFO estimator assures the lowest RMSE when the interleaved b scheme is adopted.
- For all the considered allocation schemes, a contained performance loss is observed with respect to the case of one-tap equalizion with perfect channel knowledge and perfect synchronization.

References

- T. Fusco, A. Petrella, M. Tanda: ``Non-data-aided carrier frequency offset estimation for pulse shaping OFDM/OQAM systems, '' Signal Processing, Elsevier, Vol. 88, pp. 1958-1970, August 2008.
- T. Fusco, A. Petrella, M. Tanda: ``Data aided symbol timing and CFO synchronization for filter bank multicarrier systems, '' IEEE Transactions on Wireless Communications, Vol. 8, no. 5, pp. 2705-2715, May 2009.
- T. Fusco, L. Izzo, A. Petrella, M. Tanda: ``Blind symbol timing estimation for OFDM/OQAM systems,'' IEEE Transactions on Signal Processing, accepted for publication.
- T. Fusco, A. Petrella, M. Tanda: ``Joint symbol timing and CFO estimation for OFDM/OQAM systems in multipath channels,'' Journal on Advances in Signal Processing, Special issue on Filter Banks for Next-Generation Multicarrier Wireless Communications, June 2009, submitted for publication.
- T. Fusco, A. Petrella, M. Tanda: ``Joint symbol timing and CFO estimation in multiuser OFDM/OQAM systems,'' Proc. of 10th IEEE Workshop on Signal Processing Advances in Wireless Communications, (SPAWC 2009), Perugia, Italy, June 21-24, 2009.