



# Channel Estimation and Tracking for MIMO-FBMC Systems

RA-CTI, CTTC

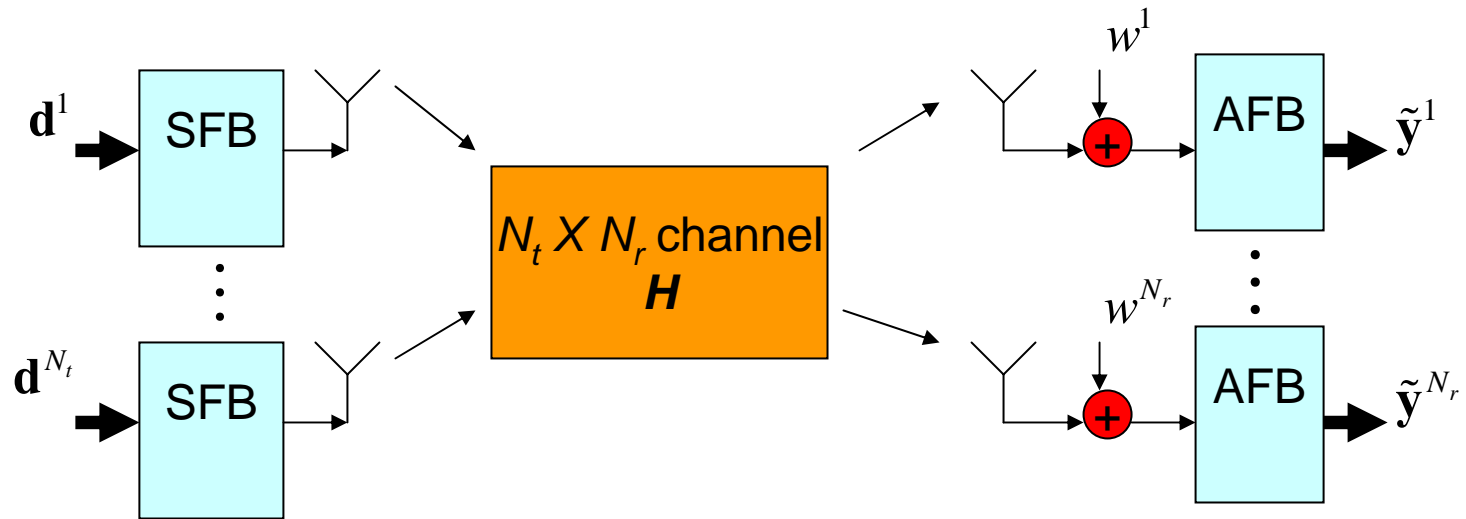
# Outline

- Introduction
  - MIMO-FBMC system
  - *Intrinsic* interference in FBMC/OQAM
- Channel Estimation
  - Preamble-based estimation
    - Full vs. Sparse preambles
    - Comparison with CP-OFDM
  - Adaptation to WiMAX DL-PUSC communication mode
- Channel Tracking
  - Kalman filtering
  - LMS-based decision-directed tracking
- Future Work



# Introduction

# MIMO-FBMC system (1)



Assumptions:

- Channel dispersion much less than pulse length
- Channel locally constant (in both time and frequency)

# MIMO-FBMC system (2)

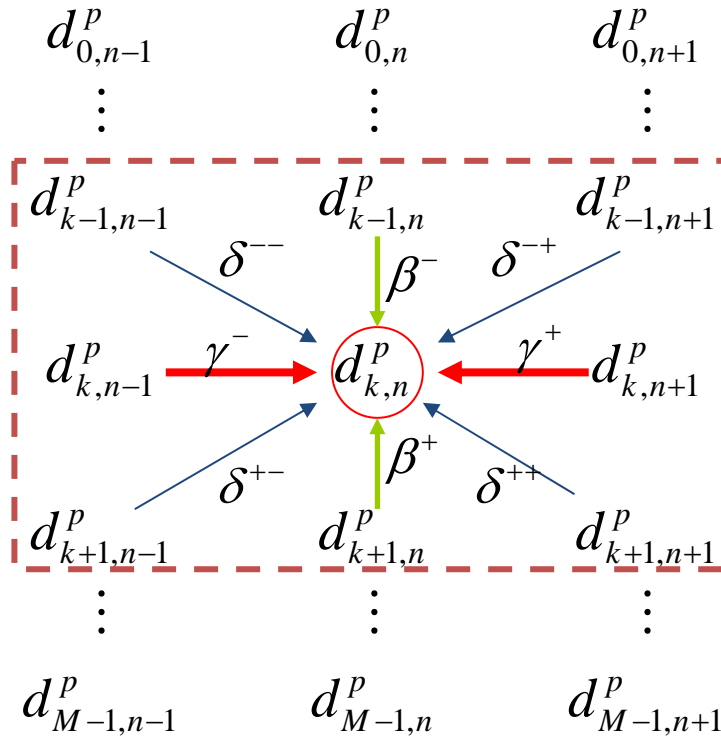
$$\underbrace{\begin{bmatrix} \tilde{y}_{k,n}^1 \\ \tilde{y}_{k,n}^2 \\ \vdots \\ \tilde{y}_{k,n}^{N_r} \end{bmatrix}}_{\tilde{\mathbf{y}}_{k,n}} = \underbrace{\begin{bmatrix} H_{k,n}^{1,1} & H_{k,n}^{2,1} & \cdots & H_{k,n}^{N_t,1} \\ H_{k,n}^{1,2} & H_{k,n}^{2,2} & \cdots & H_{k,n}^{N_t,2} \\ \vdots & \vdots & \ddots & \vdots \\ H_{k,n}^{1,N_r} & H_{k,n}^{2,N_r} & \cdots & H_{k,n}^{N_t,N_r} \end{bmatrix}}_{\mathbf{H}_{k,n}} \underbrace{\begin{bmatrix} \tilde{x}_{k,n}^1 \\ \tilde{x}_{k,n}^2 \\ \vdots \\ \tilde{x}_{k,n}^{N_t} \end{bmatrix}}_{\tilde{\mathbf{x}}_{k,n}} + \underbrace{\begin{bmatrix} \eta_{k,n}^1 \\ \eta_{k,n}^2 \\ \vdots \\ \eta_{k,n}^{N_r} \end{bmatrix}}_{\boldsymbol{\eta}_{k,n}} \Rightarrow \tilde{\mathbf{y}}_{k,n} = \mathbf{H}_{k,n} \tilde{\mathbf{x}}_{k,n} + \boldsymbol{\eta}_{k,n}$$

$H_{k,n}^{p,q}$  :  $M$ -point frequency response at TF point  $(k,n)$  for antenna pair  $(p,q)$

$$\tilde{x}_{k,n}^p = d_{k,n}^p + ju_{k,n}^p, \quad p = 1, 2, \dots, N_t$$

transmitted sequence of the  
 $p$ th antenna (data & pilots)  
(real)

intrinsic interference due to the  
neighboring time-frequency points



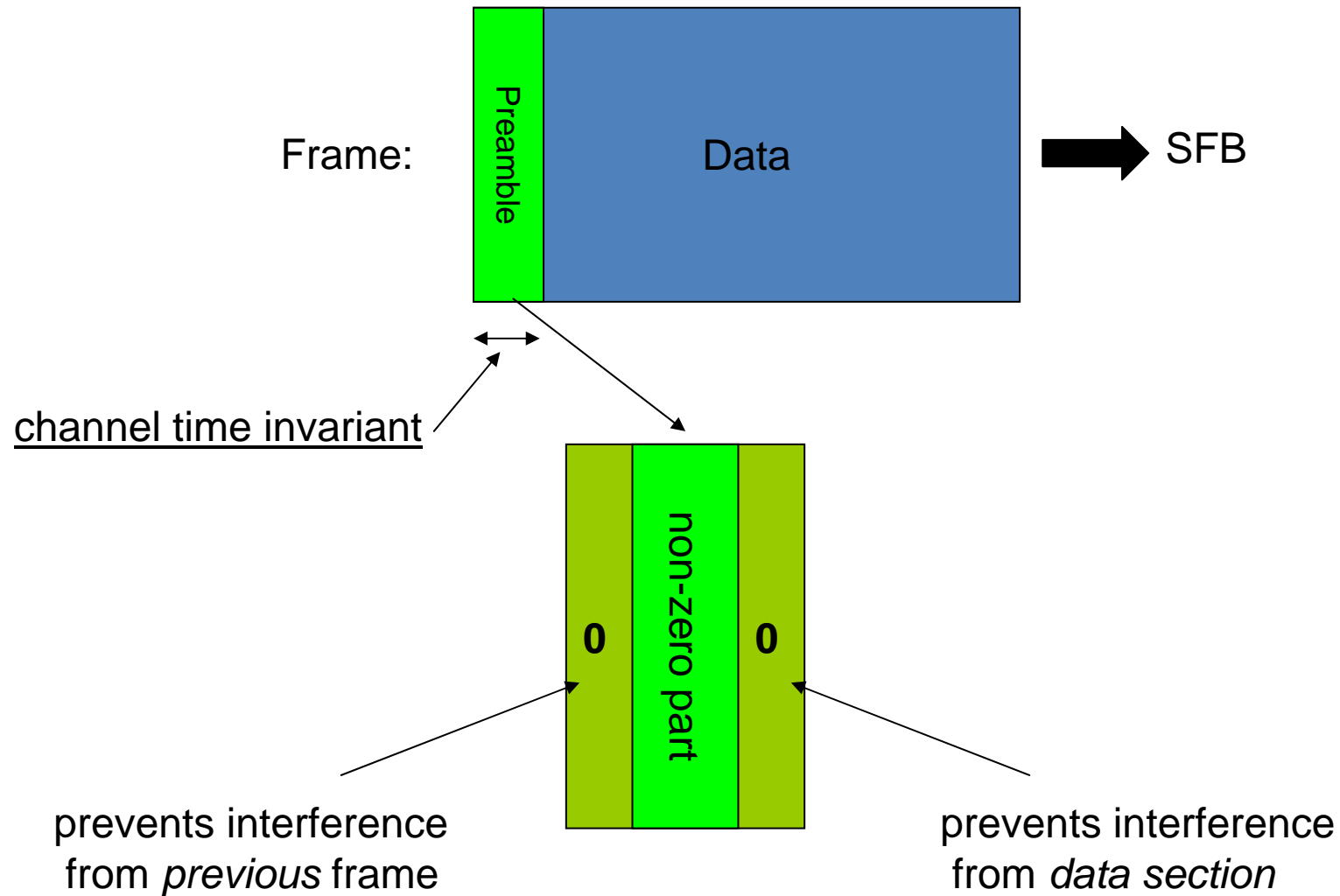
With good TF localization, contributions to intrinsic interference only come from the first-order neighboring TF points

$$\begin{aligned}
 t_{1,0} &= \beta^-, t_{-1,0} = \beta^+, \\
 t_{0,-1} &= \gamma^-, t_{0,1} = \gamma^+, \\
 t_{-1,-1} &= \delta^{--}, t_{1,-1} = \delta^{+-}, \\
 t_{-1,1} &= \delta^{-+}, t_{1,1} = \delta^{++}
 \end{aligned}$$

$$|\gamma^-| = |\gamma^+| > |\beta^-| = |\beta^+| > |\delta^{--}| = |\delta^{+-}| = |\delta^{-+}| = |\delta^{++}|$$

# Channel Estimation

# Preamble-based estimation





# Full and Sparse Preambles

*Full*  
(block-type):



Methods:  
IAM-R,  
IAM-I,  
aIAM-I

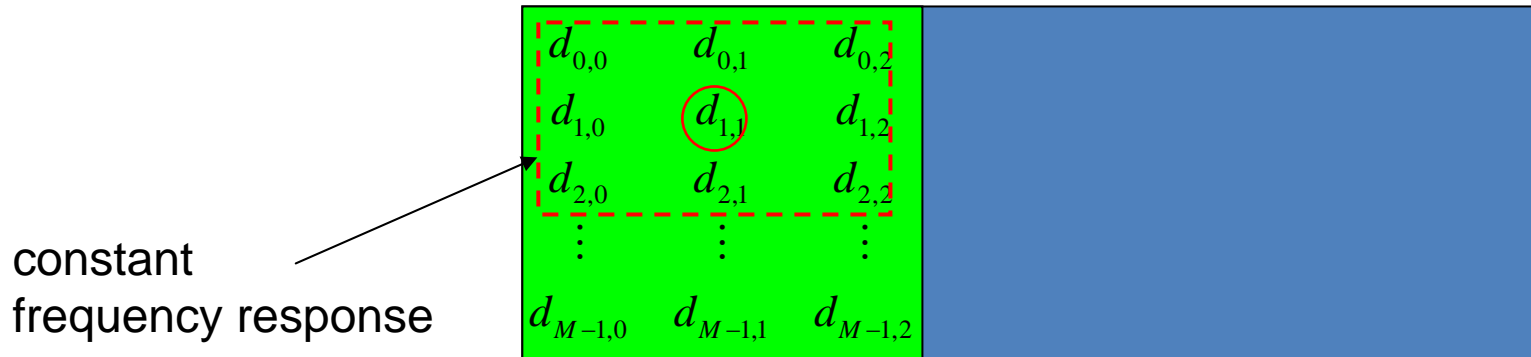
*Sparse*  
(comb-type):



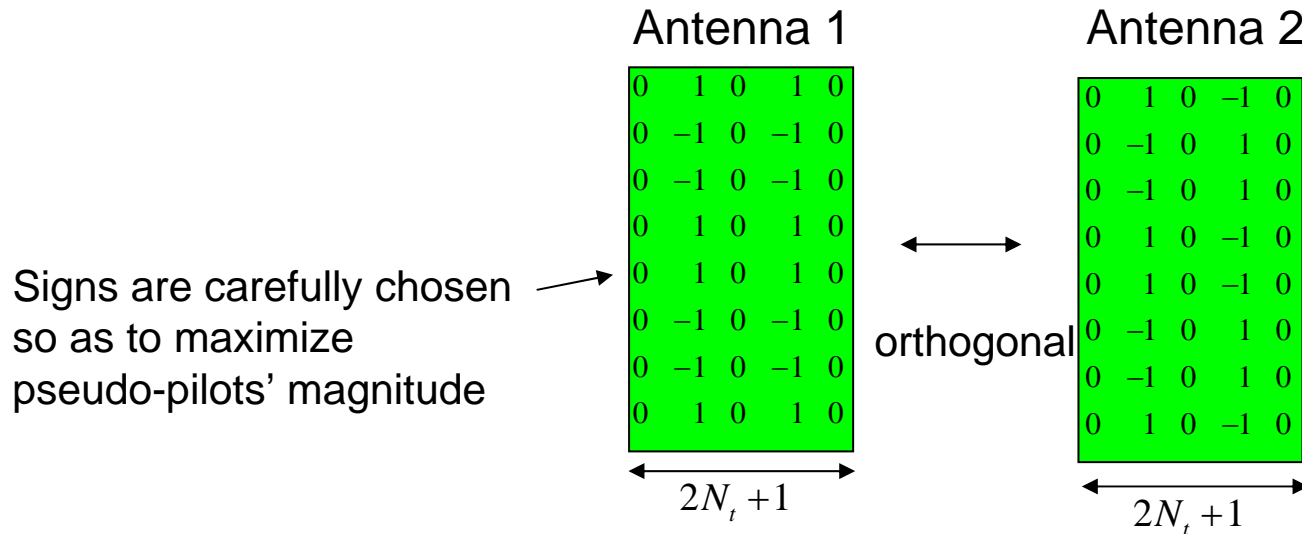
Methods:  
One carrier set,  
 $L_h$  frequencies,  
FDM pilots,  
Optimal one-symbol  
preamble

protect from ICI

# Interference Approximation Method (IAM)



- Approximate  $u_{k,1}$  at the receiver; possible if input at times 0, 2 are also known (training).
- Null 0th and 2nd symbols and choose middle symbol so as to *maximize pseudo-pilots*:  $\hat{\tilde{x}}_{k,1} = d_{k,1} + j\hat{u}_{k,1}$
- Compute channel estimate:  $\hat{H}_{k,1} = \tilde{y}_{k,1} / \hat{\tilde{x}}_{k,1}$



- Consider times 1, 3:

$$\begin{bmatrix} \tilde{\mathbf{y}}_{k,1} & \tilde{\mathbf{y}}_{k,3} \end{bmatrix} = \mathbf{H}_{k,1} \begin{bmatrix} \tilde{x}_k & \tilde{x}_k \\ \tilde{x}_k & -\tilde{x}_k \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_{k,1} & \boldsymbol{\eta}_{k,3} \end{bmatrix} = \mathbf{H}_{k,1} \tilde{x}_k \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_{k,1} & \boldsymbol{\eta}_{k,3} \end{bmatrix}$$

- Estimate channel:

$$\hat{\mathbf{H}}_{k,1} = \begin{bmatrix} \tilde{\mathbf{y}}_{k,1} & \tilde{\mathbf{y}}_{k,3} \end{bmatrix} \frac{1}{\tilde{x}_k} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} = \mathbf{H}_{k,1} + \begin{bmatrix} \boldsymbol{\eta}_{k,1} & \boldsymbol{\eta}_{k,3} \end{bmatrix} \frac{1}{2\tilde{x}_k} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

pseudo-pilot

unitary

# MIMO IAM-I

Antenna 1

0	1	0	1	0
0	-j	0	-j	0
0	-1	0	-1	0
0	j	0	j	0
0	1	0	1	0
0	-j	0	-j	0
0	-1	0	-1	0
0	j	0	j	0

Antenna 2

0	1	0	-1	0
0	-j	0	j	0
0	-1	0	1	0
0	j	0	-j	0
0	1	0	-1	0
0	-j	0	j	0
0	-1	0	1	0
0	j	0	-j	0

- Idea: Use *imaginary* pilots to create *imaginary* (or *real*) pseudo-pilots (hence of larger magnitude):

$$\hat{\tilde{x}}_{k,n} = j(d_{k,n} + \hat{u}_{k,n})$$

with  $d_{k,n}u_{k,n} > 0$ .

- Not an OQAM input!

# Averaged IAM-I (aIAM-I)

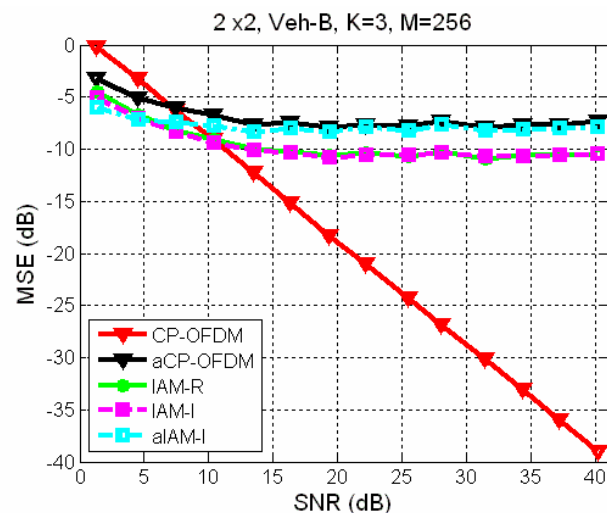
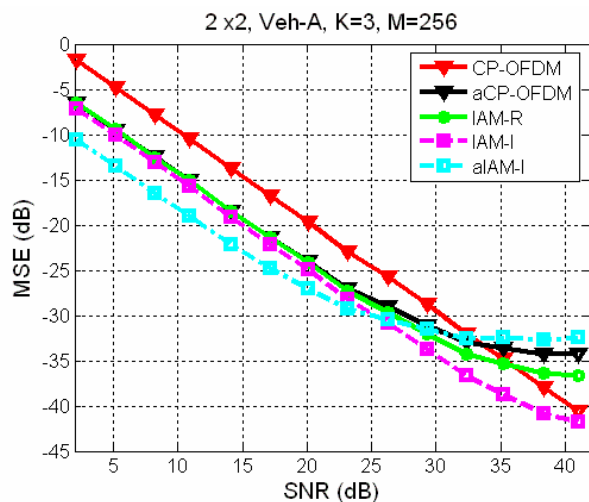
- Idea: Use IAM-I and average the resulting estimates over a small neighborhood. E.g.:

$$\hat{\mathbf{H}}_{k,n} = \frac{\hat{\mathbf{H}}_{k-1,n} + \hat{\mathbf{H}}_{k,n} + \hat{\mathbf{H}}_{k+1,n}}{3}$$

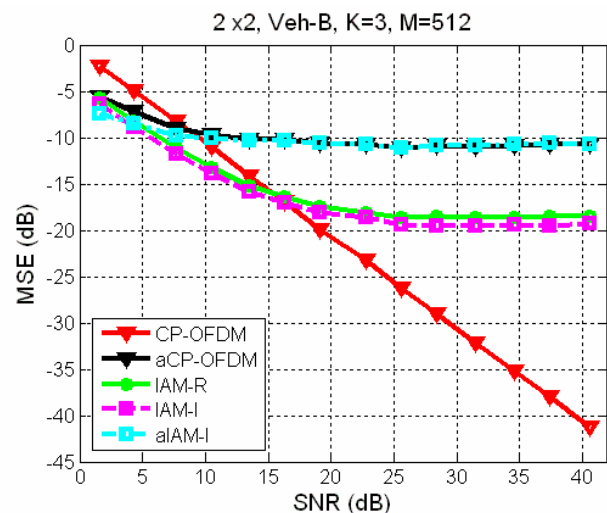
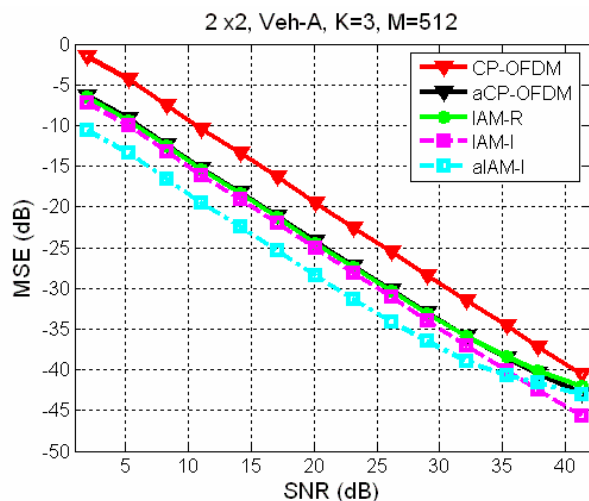
- Analogous method for CP-OFDM  $\rightarrow$  *aCP-OFDM*

# Simulation results (1)

- Simulation with the same transmit power

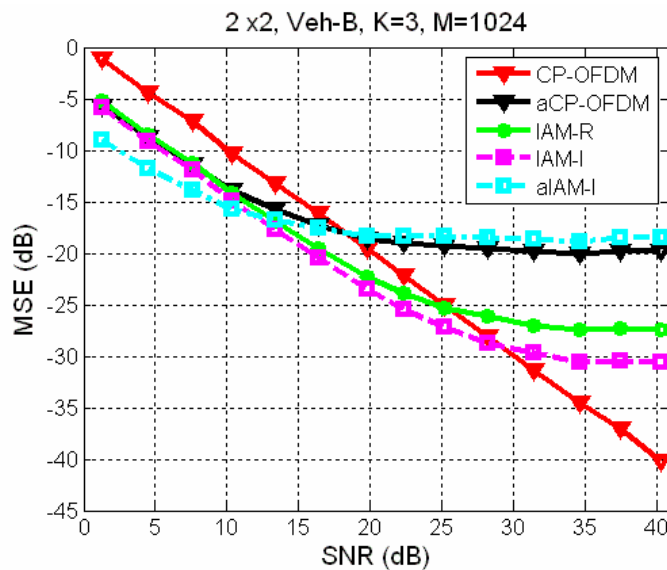
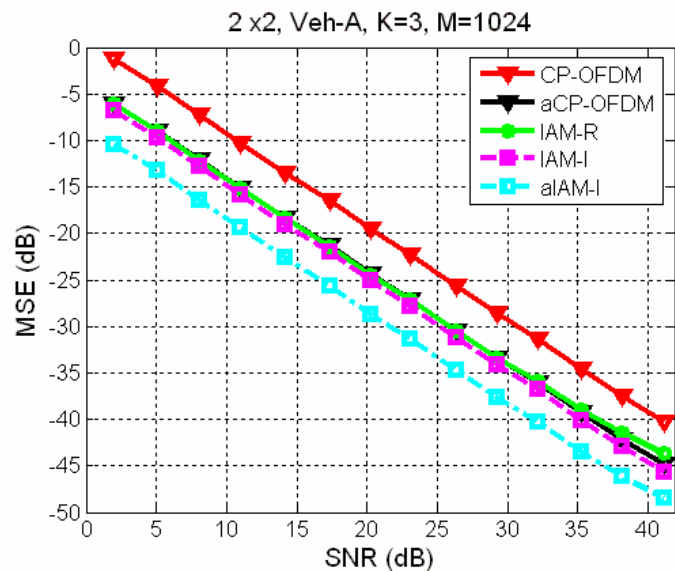


$M = 256$



$M = 512$

# Simulation results (2)



$M = 1024$

# Simulation results (3)

- IAM-based schemes perform significantly better than CP-OFDM at low and medium SNR values.
- Error floor at high SNR's due to the intrinsic interference
- Relative performance improves with more subcarriers
- aIAM-I exhibits a significant performance gain at practical SNR values in low frequency selectivity channels. When the assumption of locally flat frequency response is no longer valid, its performance deteriorates.



- One active carrier set: activate only every 3<sup>rd</sup> subcarrier ((linearly) interpolate to estimate at the rest of the frequencies)
- Use  $L_h$  pilots (equispaced & equipowered (Why?)) with DFT interpolation
- *One* nonzero symbol per antenna, with pilots frequency-division multiplexed (FDM) among the antennas

# Sparse preambles (2)

- *MSE-optimal* (for LS) *one-symbol* preamble
- $N_t$  sets of  $L_h$  equispaced and equal pilots each per antenna ( $M \gg L_h N_t$ )
- Associated  $N_t \times N_t$  matrix: unitary

$$L_h = 2, N_t = 2$$

Antenna 1

0	$x$	0
0	0	0
0	$y$	0
0	0	0
0	$x$	0
0	0	0
0	$y$	0
0	0	0

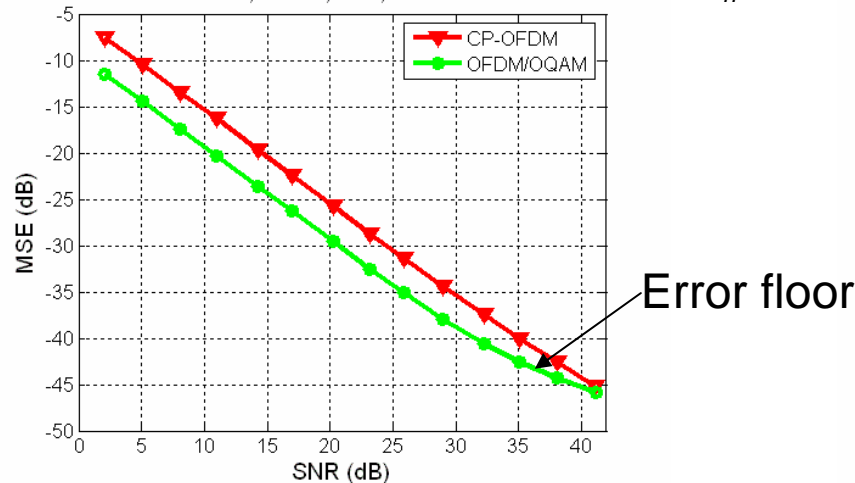
Antenna 2

0	$z$	0
0	0	0
0	$w$	0
0	0	0
0	$z$	0
0	0	0
0	$w$	0
0	0	0

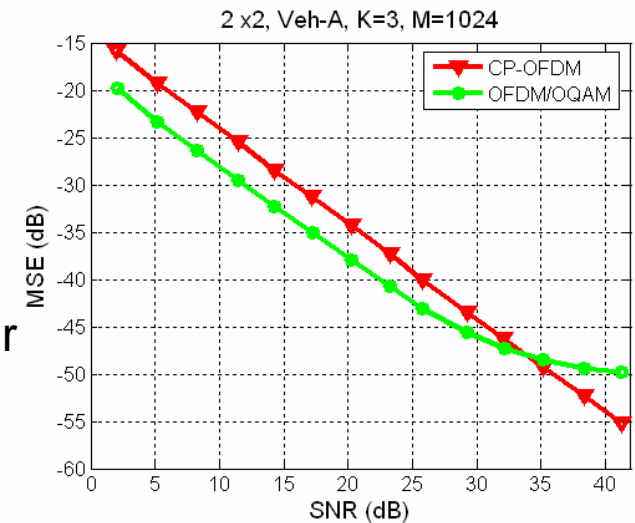
$$\begin{bmatrix} x & z \\ y & w \end{bmatrix} \text{ unitary}$$

# Simulation results

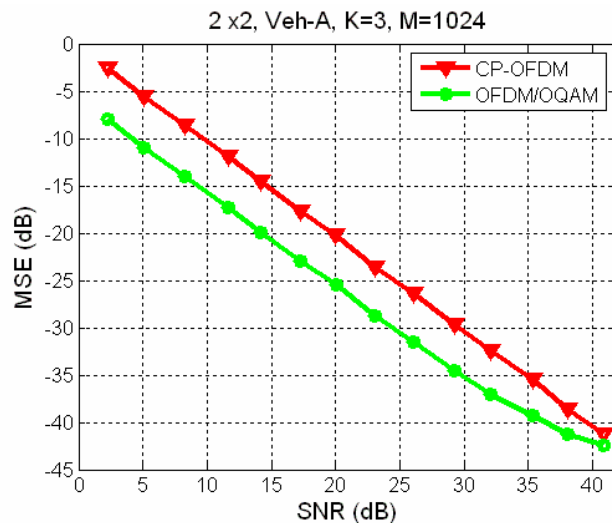
- One active carrier set 2 x 2, Veh-A, K=3, M=1024



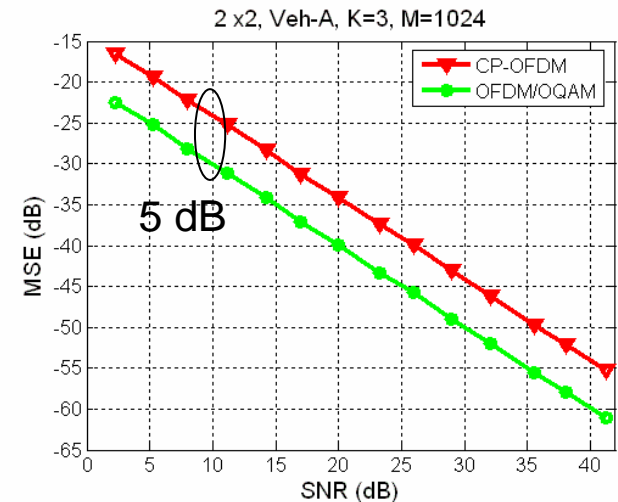
- $L_h$  Pilots



- FDM Pilots



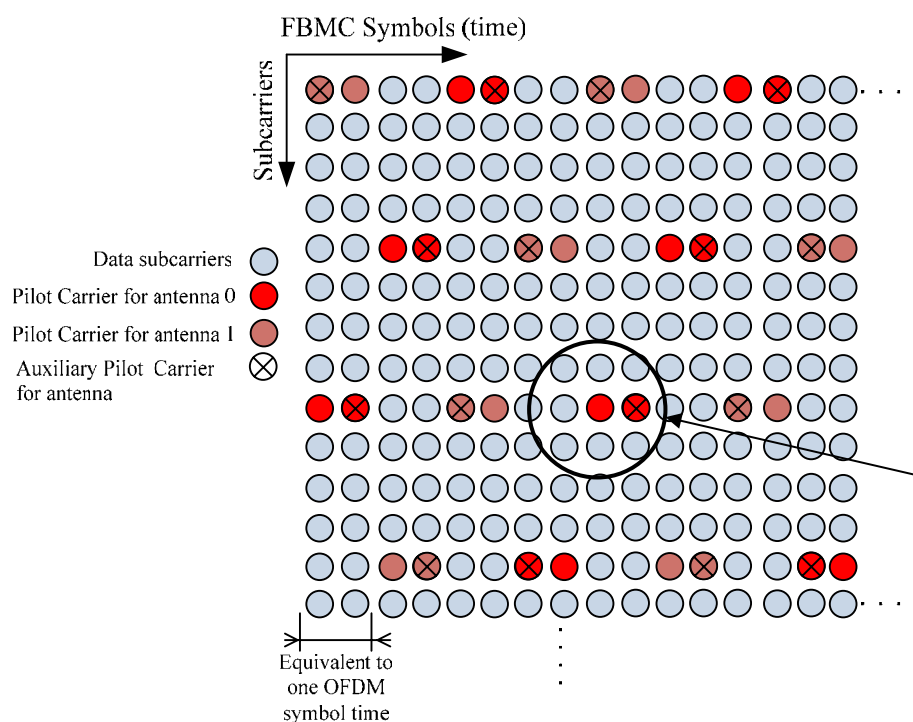
- Optimal One-Symbol



# FBMC adaptation to WiMAX DL-PUSC

## Auxiliary pilot scheme (1)

The auxiliary pilot (AP) is chosen in such a way that the secondary part (intrinsic interference) of the pilot sample becomes zero:

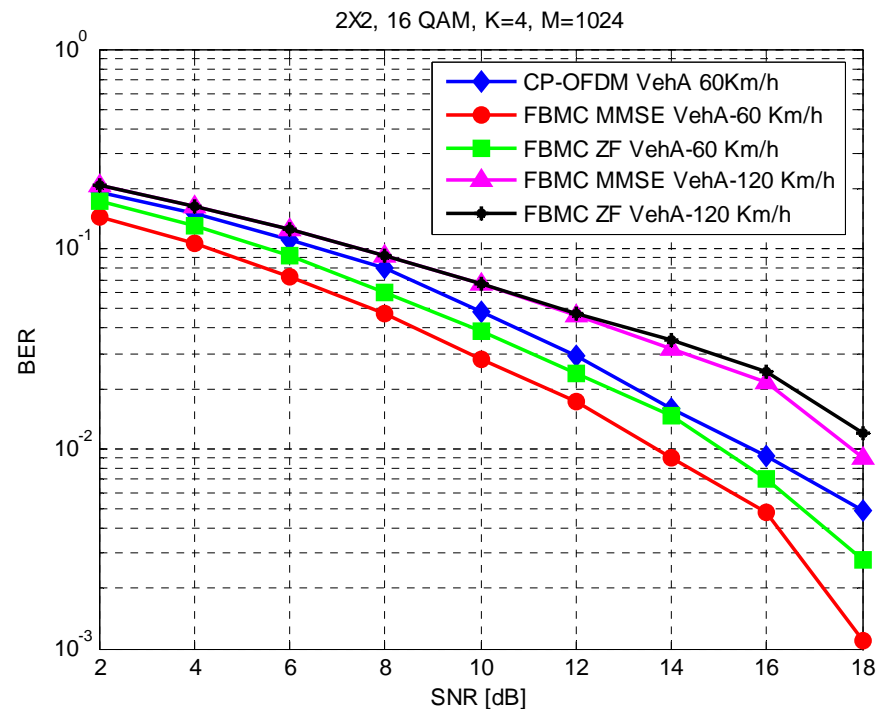


$$d_{k_a, n_a}^p = - \frac{\sum_{\substack{(k', n') \in \Omega_{k, n} \\ (k', n') \neq (k_a, n_a)}} d_{k', n'}^p t_{k-k', n-n'}}{t_{k-k_a, n-n_a}}$$

Place auxiliary pilots at the *same* subcarrier (to minimize the AP magnitude)

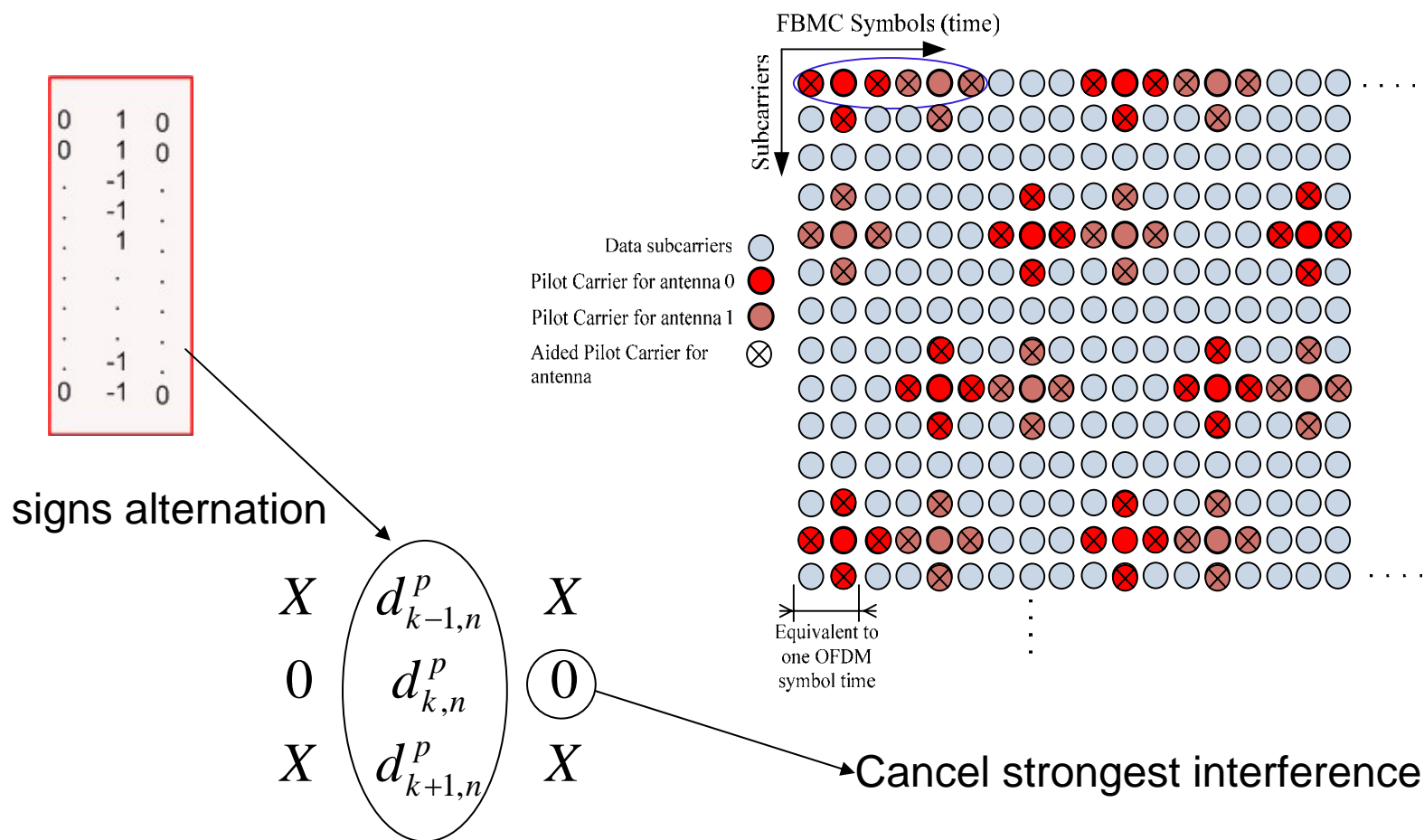
# FBMC adaptation to WiMAX DL-PUSC

## Auxiliary pilot scheme (2)

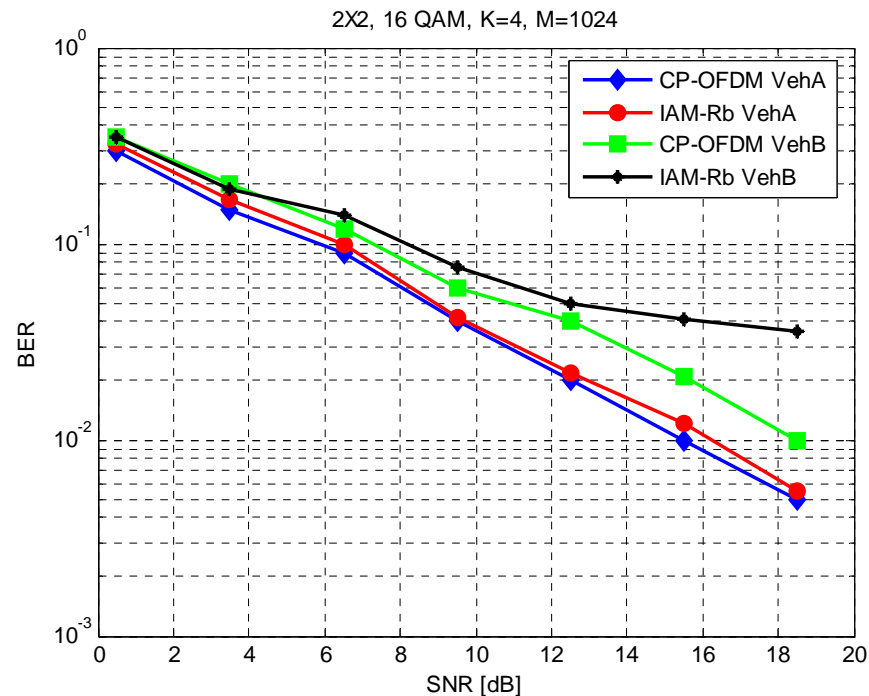


FBMC 2x2 for Veh-A channel

# Interference Approximation Method (IAM) (1)



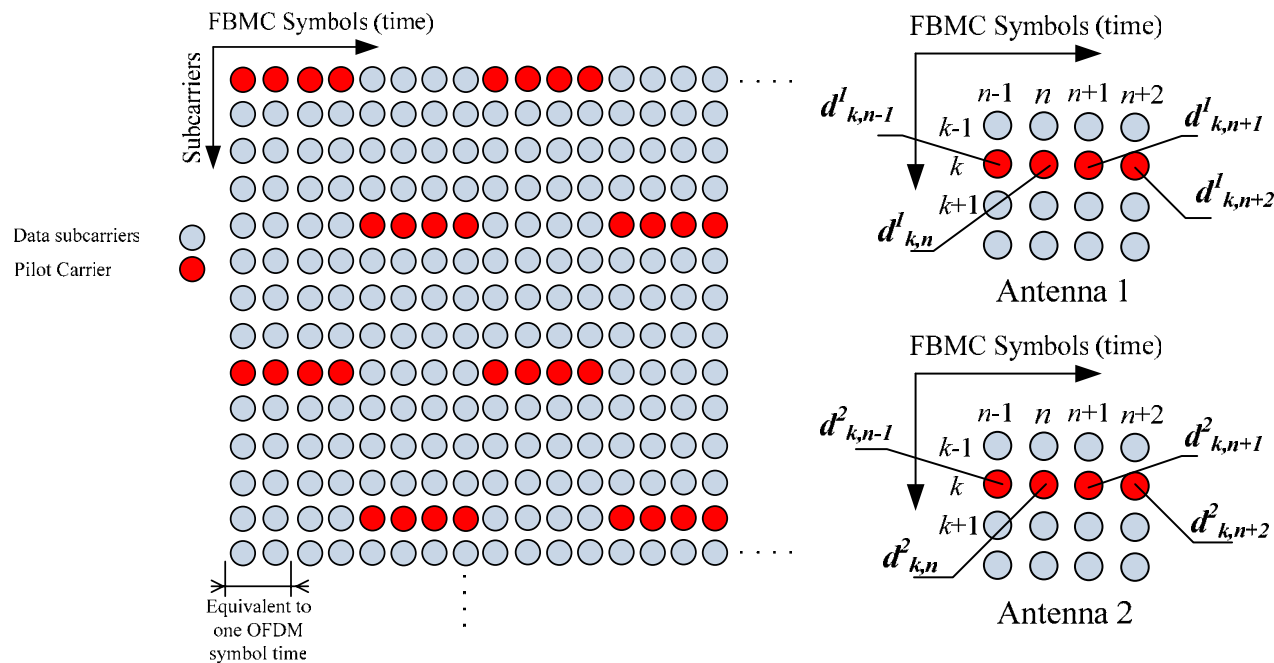
# Interference Approximation Method (IAM) (2)



FBMC 2x2 at 60 km/h using ZF equalization.

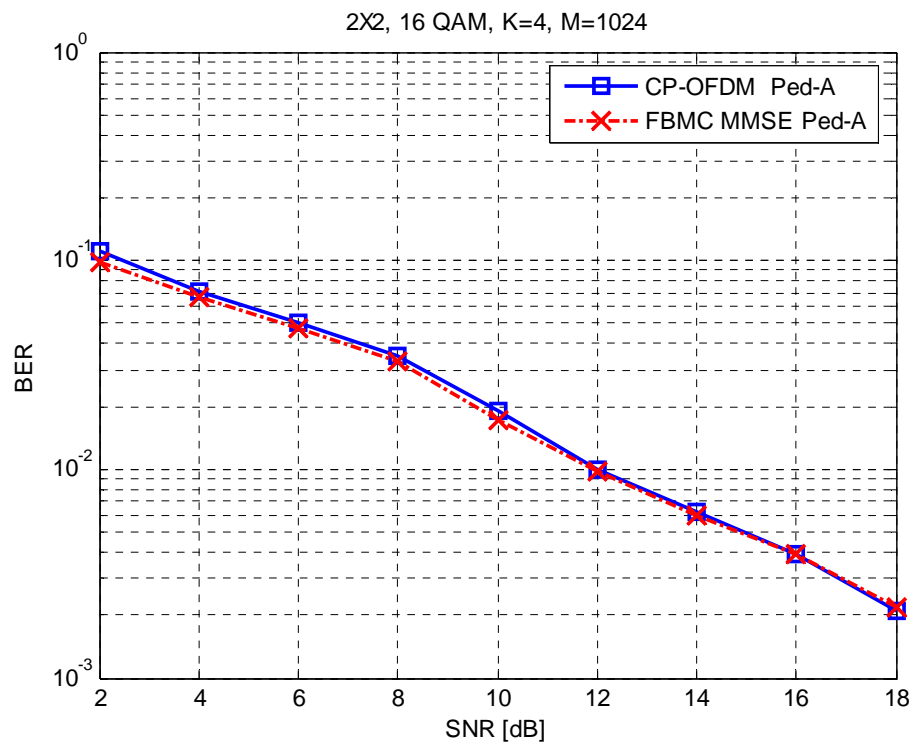
# Scattered pilots (1)

- First, the  $d_{k,n-1}^p$  symbols are determined to cancel the interference to the position  $(k,n)$ .
- Then the  $d_{k,n+2}^p$  symbols are determined to cancel the interference to the position  $(k,n+1)$ .
- Pilot pairs *orthogonal* among antennas  $\rightarrow$  compute channel by solving a 2x2 system of equations





# Scattered pilots (2)



Simplified orthogonal pilot sequence in FBMC MIMO system with 2 transmit antennas

SDM case over Ped-A channel with non iterative MMSE receiver

# Channel Tracking

# Channel tracking

- User mobility and typical channel coherence times (e.g., coherence time of 2 ms (100 km/h) vs. frame duration of 5 ms in WiMAX)
- Channel remains invariant in the duration of (at least) an FBMC symbol.
- Based on a channel variation model:
  - Kalman filtering (with measurement and state-noise covariances)
  - Frame-by-frame (coarse scale) or symbol-by-symbol (fine scale) tracking (allocated training pilots)
  - Complexity issue → for limited mobility
- No channel variation model assumed
  - Decision-directed (no pilots) LMS for symbol-by-symbol tracking
  - Low complexity

# Kalman filtering (1)

- State equation (AR(1) model):

$$\mathbf{H}[\zeta] = a\mathbf{H}[\zeta - 1] + \mathbf{v}[\zeta]$$

- Measurement equation:

$$\mathbf{y}[\zeta] = \mathbf{D}[\zeta]\mathbf{H}[\zeta] + \mathbf{w}[\zeta]$$

- Process noise covariance:  $\mathbf{Q}[\zeta]$
- Measurement noise covariance:  $\mathbf{R}[\zeta]$

## Kalman filtering (2)

$$\hat{\mathbf{H}}[\zeta + 1|\zeta] = a\hat{\mathbf{H}}[\zeta|\zeta]$$

$$\mathbf{P}[\zeta + 1|\zeta] = |a|^2 \mathbf{P}[\zeta|\zeta] + \mathbf{Q}[\zeta]$$

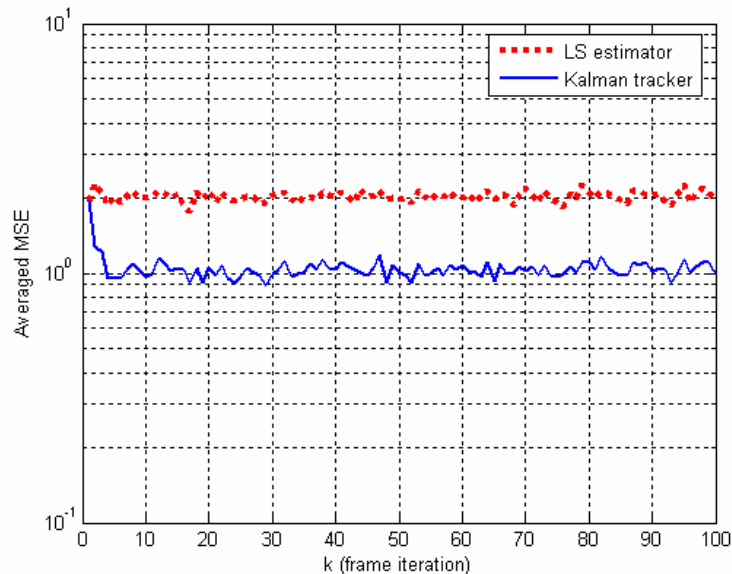
$$\mathbf{K}[\zeta + 1] = a\mathbf{P}[\zeta + 1|\zeta]\mathbf{D}[\zeta + 1]^H \left( \mathbf{D}[\zeta + 1]\mathbf{P}[\zeta + 1|\zeta]\mathbf{D}[\zeta + 1]^H + \mathbf{R}[\zeta + 1] \right)^{-1}$$

$$\mathbf{P}[\zeta + 1|\zeta + 1] = (\mathbf{I} - a\mathbf{K}[\zeta + 1]\mathbf{D}[\zeta + 1])\mathbf{P}[\zeta + 1|\zeta]$$

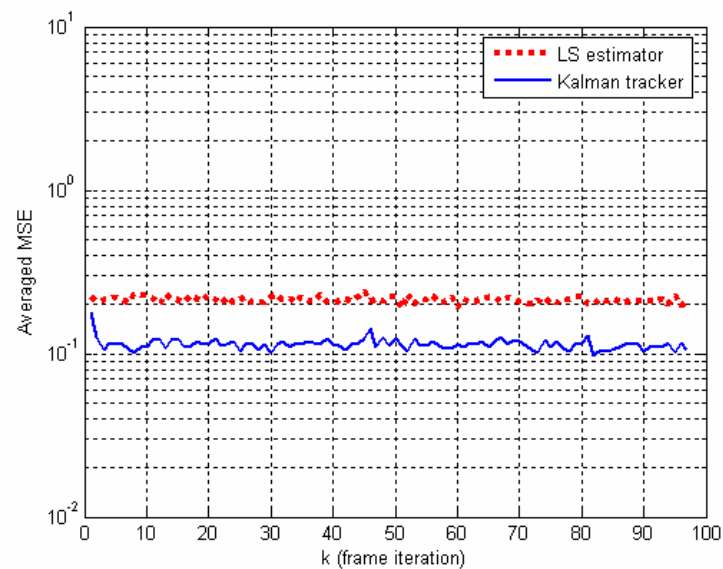
$$\mathbf{e}[\zeta + 1] = \mathbf{y}[\zeta + 1] - \mathbf{D}[\zeta + 1]\hat{\mathbf{H}}[\zeta + 1|\zeta]$$

$$\hat{\mathbf{H}}[\zeta + 1|\zeta + 1] = \hat{\mathbf{H}}[\zeta + 1|\zeta] + \mathbf{K}[\zeta + 1]\mathbf{e}[\zeta + 1]$$

# Kalman filtering (3)



SNR = 0 dB



SNR = 10 dB

- MISO system
- frame-by-frame tracking
- 3 dB gain in MSE over no tracking

# LMS-based decision-directed tracking (1)

$$\mathbf{h}_n^q = \left[ \left( \mathbf{h}_n^{1,q} \right)^T \quad \left( \mathbf{h}_n^{2,q} \right)^T \quad \dots \quad \left( \mathbf{h}_n^{N_t,q} \right)^T \right]^T$$

$$\mathbf{h}_n = \left[ \left( \mathbf{h}_n^1 \right)^T \quad \left( \mathbf{h}_n^2 \right)^T \quad \dots \quad \left( \mathbf{h}_n^{N_r} \right)^T \right]^T$$

$$\mathbf{D}_n^p = \text{diag} \left( x_{k_0,n}^p, x_{k_1,n}^p, \dots, x_{k_{P-1},n}^p \right)$$

$$\mathbf{T}_n = \left[ \mathbf{D}_n^1 \mathbf{F}_{P \times L_h} \quad \mathbf{D}_n^2 \mathbf{F}_{P \times L_h} \quad \dots \quad \mathbf{D}_n^{N_t} \mathbf{F}_{P \times L_h} \right]$$

$$\mathbf{y}_n = \left[ \left( \mathbf{y}_n^1 \right)^T \quad \left( \mathbf{y}_n^2 \right)^T \quad \dots \quad \left( \mathbf{y}_n^{N_r} \right)^T \right]^T$$

$$\mathbf{y}_n = \underbrace{\left( \mathbf{I}_{M_R} \otimes \mathbf{T}_n \right)}_{\mathbf{S}_n} \mathbf{h}_n + \boldsymbol{\eta}_n = \mathbf{S}_n \mathbf{h}_n + \boldsymbol{\eta}_n$$

# LMS-based decision-directed tracking (2)

$$\hat{\mathbf{h}}_{n-1} \xrightarrow{\text{DFT}} \hat{\mathbf{H}}_{k,n-1}, \quad k \in \text{selected subcarriers}$$

$$\hat{\mathbf{x}}_{k,n} = \hat{\mathbf{H}}_{k,n-1}^+ \mathbf{y}_{k,n} \quad (\text{or some other equalizer (MMSE, V-BLAST, etc.)})$$

$$\hat{\mathbf{x}}_{k,n} = \hat{\mathbf{d}}_{k,n} + j\hat{\mathbf{u}}_{k,n} = \text{dec} \left[ \text{Re}(\hat{\mathbf{x}}_{k,n}) \right] + j \text{Im}(\hat{\mathbf{x}}_{k,n})$$

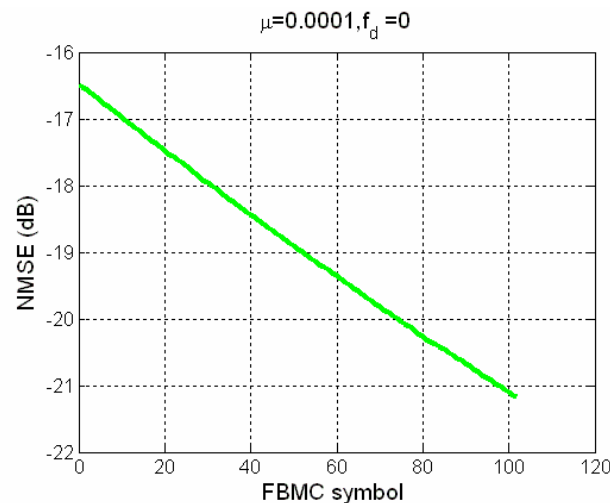
$$\hat{\mathbf{x}}_{k,n} \rightarrow \hat{\mathbf{D}}_n \rightarrow \hat{\mathbf{S}}_n$$

$$\mathbf{e}_n = \mathbf{y}_n - \hat{\mathbf{S}}_n \hat{\mathbf{h}}_{n-1}$$

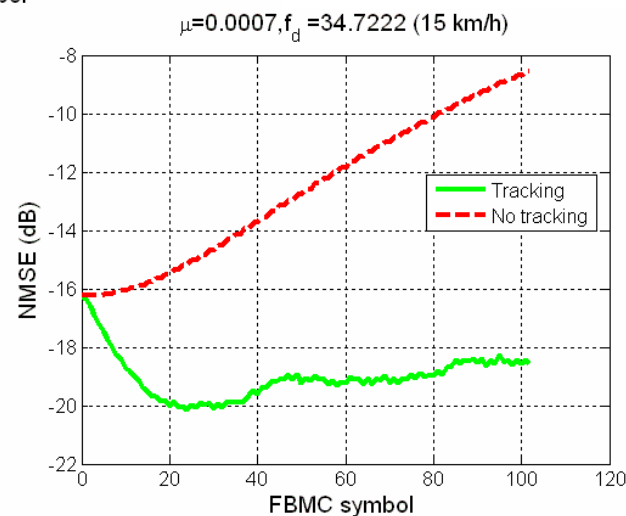
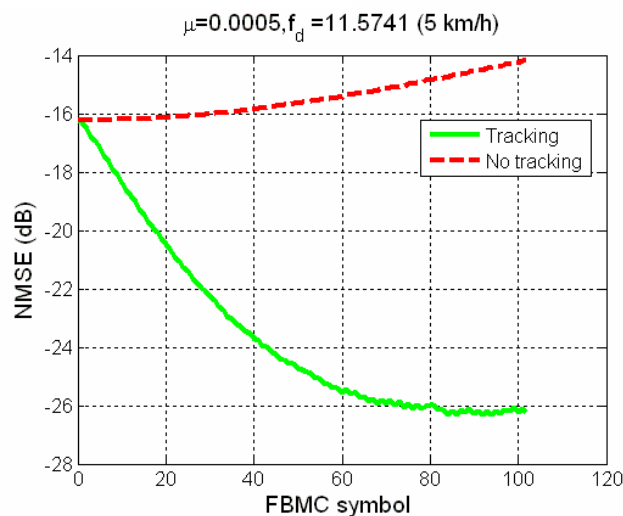
$$\hat{\mathbf{h}}_n = \hat{\mathbf{h}}_{n-1} + \mu \hat{\mathbf{S}}_n^H \mathbf{e}_n \quad \mu: \text{step size}$$



# LMS-based decision-directed tracking (3)



SNR = 20 dB,  
M = 512, K = 4



# References

- J.-P. Javardin *et al.*, "Pilot-aided channel estimation for OFDM/OQAM," *VTC '03 (Spring)*.
- C. L     *et al.*, "Channel estimation methods for preamble-based OFDM/OQAM modulations," *EW '07*.
- C. L     *et al.*, "2 dB better than CP-OFDM with OFDM/OQAM for preamble-based channel estimation," *ICC '08*.
- J.-P. Javardin and Y. Jiang, "Channel estimation in MIMO OFDM/OQAM," *SPAWC '08*.
- R. Negi and J. Cioffi, "Pilot tone selection for channel estimation in a mobile OFDM system," *IEEE Trans. Consumer Electronics*, Aug. 1998.
- Z. Wu *et al.*, "Design of optimal pilot-tones for channel estimation in MIMO-OFDM systems," *WCNC '05*.
- C. Zhenlan and D. Dahlhaus, "Time versus frequency domain channel tracking using Kalman filters for OFDM systems with antenna arrays," *VTC '03 (Spring)*.

## Future Work

- Refining tracking results (e.g., step size selection, include scattered pilots)
- Extension to multi-user MIMO-FBMC
- Adaptation of existing MIMO equalization techniques (e.g., V-BLAST, sphere decoding) to MIMO-FBMC
- Preamble design for *joint* channel and CFO estimation