MMSE equalization of FBMC systems with forward error correction

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Outline

1. Linear Equalization of FBMCs
2. Decision Feedback Equalization of FBMCs
3. Conclusions
Linear MMSE Equalization of FBMC Systems

Subchannel $k$:

$Q$: Length of the impulse responses $g'_k[n], m'_k[n]$ and $n'_k[n]$.

$N$: Equalizer $w_k[n]$ length.
Linear MMSE Equalization of FBMC Systems

Optimization problem:

$$w_{k,MMSE} = \arg \min_{w_k} E \left[ |\hat{a}_k[m] - a_k[m - \nu]|^2 \right],$$

where

$$\hat{a}_k[m] = \text{Re} \left( w_k^H y_k[n] \right), \quad n = 2m, \quad m \in \mathbb{Z},$$

$\nu$ is the equalization delay and

$$y_k[n] = [y_k[n], \ y_k[n - 1], \ ..., \ y_k[n - N + 1]]^T \in \mathbb{C}^N.$$
Linear MMSE Equalization of FBMC Systems
Solution for i.i.d. input symbols $d_k[m]$ with variance $\sigma_d^2$:

$$ w'_{k,\text{MMSE}} = \left[H_kH_k^T + F_kF_k^T + R_{\eta,k}\right]^{-1} \frac{\sigma_d}{\sqrt{2}} H_k e_\nu, $$

with $H_k = \frac{\sigma_d}{\sqrt{2}} \begin{bmatrix} G_k^{(R)} \\ G_k^{(I)} \end{bmatrix}$, $F_k = \frac{\sigma_d}{\sqrt{2}} \begin{bmatrix} M_k^{(R)} & N_k^{(R)} \\ M_k^{(I)} & N_k^{(I)} \end{bmatrix}$,

$$ R_{\eta,k} = \frac{\sigma_\eta^2}{2} \Gamma'_k \Gamma'_{k,T}, \quad \Gamma'_k = \begin{bmatrix} \Gamma_k^{(R)} \\ \Gamma_k^{(I)} \end{bmatrix}, $$

$G_k^{(R)}, G_k^{(I)}, M_k^{(R)}, M_k^{(I)}, N_k^{(R)}, N_k^{(I)}, \Gamma_k^{(R)}, \Gamma_k^{(I)}$: Real and imaginary parts of the convolution matrices obtained from $g'_k[n], m'_k[n], n'_k[n]$ and $\gamma'_k[n]$.

e_\nu: \nu\text{-th unit vector.}

Waldhauser, D.S.; Baltar, L.G.; Nossek, J.A.; MMSE subcarrier equalization for filter bank based multicarrier systems; SPAWC 2008.
Adaptive Linear MMSE Equalization of FBMC Systems

O-QAM LMS:

\[ w_k[n] \]

\[ \Delta w_k[n] \]

\[ e_k[n] \]

\[ \epsilon_k[m] \]

\[ O_k \]

\[ O'_k \]

\[ \hat{d}_k[m] \]

\[ \tilde{d}_k[m] \]

Waldhauser, D.S.; Baltar, L.G.; Nossek, J.A.; Adaptive equalization for filter bank based multicarrier systems; ISCAS 2008.
Noise correlations

- Three distinguishable cases: Intra-subcarrier, Adjacent inter-subcarrier, Non-adjacent inter-subcarrier
  - Ideal scenario (frequency flat channel and no equalizer)
    - Intra-subcarrier - Non existent
    - Adjacent inter-subcarrier - Non existent
    - Non-adjacent inter-subcarrier - Non existent
  - Low frequency selective channel and one-tap equalizer
    - Intra-subcarrier - Non existent
    - Adjacent inter-subcarrier - small
    - Non-adjacent inter-subcarrier - very small (prototype dependent)
  - High frequency selective channel and linear multitap equalizer
    - Intra-subcarrier - expected
    - Adjacent inter-subcarrier - expected
    - Non-adjacent inter-subcarrier - very small (prototype dependent)
Noise correlations

- Correlations may affect the coded BER performance
- A WiMAX simulation scenario was considered and the coded BER was evaluated
- Channel coding with and without interleaving was considered
- Convolutional encoder with native rate 1/2, constraint length 7 and generator polynomials (IEEE Std. 802.16-2004): \( g_1(D) = 1 + D + D^2 + D^3 + D^6 \) and \( g_2(D) = 1 + D^2 + D^3 + D^5 + D^6 \)
- Block interleaver of length \( 2^b \times M_{\text{used}} \) defined in IEEE Std. 802.16-2004
- A soft decoder (approximate LLR, “unquantized” inputs to the Viterbi decoder)
Coded BER Comparison between FBMC and CP-OFDM

Linear MMSE Eq. in WiMAX scenario

Parameters:
- 16-QAM
- $C_r = 1/2$
- Conv. code
- Soft decod.
- $M = 1024$
- $M_{\text{data}} = 768$
- $\Delta f = 10.9 \text{ kHz}$
- $BW = 10 \text{ MHz}$
- $T_s = 89.28 \text{ ns}$
- $T_{cp} = 91.42 \mu s (1/4)$
- $N = 7$
- ITU Veh. B Static
- $\tau_{\text{RMS}} = 4 \mu s$
MMSE Decision Feedback Equalizer for FBMC Systems

Per-subchannel DFE and OQAM de-staggering $O_k^\prime$, $k$ even:

![Diagram](image.png)
MMSE Decision Feedback Equalizer for FBMC Systems

Optimization problem:

\[
\begin{align*}
(w_{k,\text{MMSE}}, f_{k,\text{MMSE}}) &= \arg \min_{(w_k, f_k)} \mathbb{E} \left[ |\hat{a}_k[m] - a_k[m - \nu]|^2 \right], \\
\text{where} \\
\hat{a}_k[m] &= \text{Re} \left[ w_k^H y_k[n] \right], \quad n = 2m, \quad m \in \mathbb{Z}, \\
\nu \text{ is the equalization delay and} \\
y_k[n] &= [y_k[n], y_k[n - 1], \ldots, y_k[n - N + 1]]^T \in \mathbb{C}^N.
\end{align*}
\]
MMSE Decision Feedback Equalizer for FBMC Systems

DFE solution for i.i.d. input symbols $d_k[m]$ with variance $\sigma_d^2$:

$$
\begin{bmatrix}
  w'_{k,\text{MMSE}} \\
  f_{k,\text{MMSE}}(R)
\end{bmatrix} = 
\begin{bmatrix}
  H_k H_k^T + F_k F_k^T + R_{\eta,k} - \frac{\sigma_d}{\sqrt{2}} H_k J_\nu \\
  - \frac{\sigma_d}{\sqrt{2}} J_\nu^T H_k^T \\
  - \frac{\sigma_d^2}{2} I_{B+1}
\end{bmatrix}^{-1} 
\begin{bmatrix}
  \frac{\sigma_d}{\sqrt{2}} H_k e_\nu \\
  0_{B+1}
\end{bmatrix},
$$

where

$$
J_\nu = \begin{cases}
  \begin{bmatrix} 0_{(\nu+1)\times(B+1)} & I_{B+1} & 0_{(L-B-\nu-1)\times(B+1)} \end{bmatrix}^T, & L - \nu > B + 1, \\
  \begin{bmatrix} 0_{(\nu+1)\times(B+1)} & I_{B+1} \end{bmatrix}^T, & L - \nu < B + 1, \\
  \begin{bmatrix} 0_{(\nu+1)\times(B+1)} & I_{B+1} \end{bmatrix}^T, & L - \nu = B + 1.
\end{cases}
$$

$B$: length of the feedback filter

$L = N + Q - 2$

Baltar, L.G.; Waldhauser, D.S.; Nossek, J.A.; MMSE subchannel decision feedback equalization for filter bank based multicarrier systems; ISCAS 2009.
BER Comparison between FBMC and CP-OFDM

MMSE DFE in WiMAX scenario:

Parameters:

16-QAM
\( M = 1024(256) \)
\( M_{\text{data}} = 840(210) \)
\( \Delta f = 10.9(43.7) \text{ kHz} \)
\( BW = 10 \text{ MHz} \)
\( T_s = 89.28 \text{ ns} \)
\( T_{cp} = 91.42 \mu s(1/4) \)
Extended TU 3GPP
Static
\( \tau_{\text{RMS}} = 0.99 \mu s \)
Adaptive MMSE DF Equalizer for FBMC Systems

Per-subchannel decision directed OQAM adaptive DFE:

\[
y_k[n] \xrightarrow{w_k[n]} \Delta w_k[n], \Delta f_k[n] \xrightarrow{LMS} \hat{w}_k[n] \xrightarrow{f_k[n]} \hat{x}_k[n] \xrightarrow{O_k} \epsilon_k[n] \xrightarrow{O_k} \epsilon_k[n] \xrightarrow{\hat{d}_k[m]} \tilde{d}_k[m]
\]

Waldhauser, D.S.; Baltar, L.G.; Nossek, J.A.; *Adaptive decision feedback equalization for filter bank based multicarrier systems*; ISCAS 2009.
Adaptive DF Equalizer for FBMC Systems

Adaptive O-QAM DFE LMS in a static WiMAX scenario:

Parameters:

16-QAM

- \( M = 1024(256) \)
- \( M_{\text{used}} = 840(210) \)
- \( \Delta f = 10.9(43.7) \text{ kHz} \)
- \( B = 10 \text{ MHz} \)
- \( T_s = 89.28 \text{ ns} \)
- \( T_{cp} = 91.42 \mu s(1/4) \)
- ITU Veh. A Static
- \( \tau_{\text{RMS}} = 0.37 \mu s \)
Conclusions

- The effect of noise correlations in a coded FBMC system is negligible for a WiMAX scenario.
- DFE should be employed only when the frequency selectivity inside one subcarrier is very high.
- The equalizer analytical solutions are based on perfect channel knowledge.