MMSE equalization of FBMC systems with forward error correction

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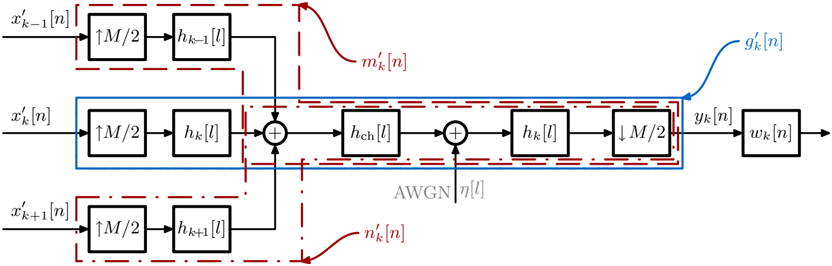
June 21, 2009

Outline

Linear MMSE Equalization of FBMC Systems

Subchannel k:



Q: Length of the inpulse responses $g'_k[n]$,$m'_k[n]$ and $n'_k[n]$.
N: Equalizer $w_k[n]$ length.

Linear MMSE Equalization of FBMC Systems

Optimization problem:

$$\boldsymbol{w}_{k,\mathsf{MMSE}} = \arg\min_{\boldsymbol{w}_k} \mathsf{E}\left[\left|\hat{a}_k[m] - a_k[m-\nu]\right|^2\right],$$

where

$$\hat{a}_k[m] = \mathsf{Re}\left[\boldsymbol{w}_k^{\mathsf{H}} \boldsymbol{y}_k[n]\right], \quad n = 2m, \quad m \in \mathbb{Z},$$

ν is the equalization delay and
$\boldsymbol{y}_k[n] = [y_k[n],\ y_k[n-1],\ \ldots,\ y_k[n-N+1]]^T \in \mathbb{C}^N$.

Linear MMSE Equalization of FBMC Systems

Solution for i.i.d. input symbols $d_k[m]$ with variance σ_d^2:

$$\boxed{\boldsymbol{w}'_{k,\mathrm{MMSE}} = \left[\boldsymbol{H}_k\boldsymbol{H}_k^{\mathsf{T}} + \boldsymbol{F}_k\boldsymbol{F}_k^{\mathsf{T}} + \boldsymbol{R}_{\eta,k}\right]^{-1} \frac{\sigma_d}{\sqrt{2}}\boldsymbol{H}_k\boldsymbol{e}_\nu,}$$

with
$$\boldsymbol{H}_k = \frac{\sigma_d}{\sqrt{2}}\begin{bmatrix}\boldsymbol{G}_k^{(\mathrm{R})}\\ \boldsymbol{G}_k^{(\mathrm{I})}\end{bmatrix}, \qquad \boldsymbol{F}_k = \frac{\sigma_d}{\sqrt{2}}\begin{bmatrix}\boldsymbol{M}_k^{(\mathrm{R})} & \boldsymbol{N}_k^{(\mathrm{R})}\\ \boldsymbol{M}_k^{(\mathrm{I})} & \boldsymbol{N}_k^{(\mathrm{I})}\end{bmatrix},$$

$$\boldsymbol{R}_{\eta,k} = \frac{\sigma_\eta^2}{2}\boldsymbol{\Gamma}'_k\boldsymbol{\Gamma}'^{,T}_k, \qquad \boldsymbol{\Gamma}'_k = \begin{bmatrix}\boldsymbol{\Gamma}_k^{(R)} & \boldsymbol{\Gamma}_k^{(I)}\\ \boldsymbol{\Gamma}_k^{(I)} & -\boldsymbol{\Gamma}_k^{(R)}\end{bmatrix},$$

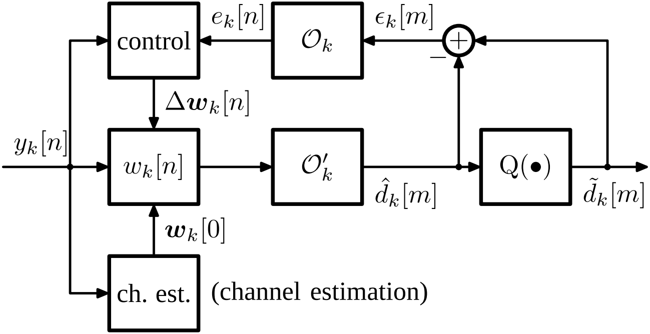
$\boldsymbol{G}_k^{(\mathrm{R})}$, $\boldsymbol{G}_k^{(\mathrm{I})}$ $\boldsymbol{M}_k^{(\mathrm{R})}$, $\boldsymbol{M}_k^{(\mathrm{I})}$, $\boldsymbol{N}_k^{(\mathrm{R})}$, $\boldsymbol{N}_k^{(\mathrm{I})}$, $\boldsymbol{\Gamma}_k^{(\mathrm{R})}$, $\boldsymbol{\Gamma}_k^{(\mathrm{I})}$: Real and imaginary parts of the convolution matrices obtained from $g'_k[n]$,$m'_k[n]$, $n'_k[n]$ and $\gamma'_k[n]$.

$\boldsymbol{e}_\nu$: ν-th unit vector.

Waldhauser, D.S.; Baltar, L.G.; Nossek, J.A.; *MMSE subcarrier equalization for filter bank based multicarrier systems*; SPAWC 2008.

Adaptive Linear MMSE Equalization of FBMC Systems

O-QAM LMS:



Waldhauser, D.S.; Baltar, L.G.; Nossek, J.A.; *Adaptive equalization for filter bank based multicarrier systems*; ISCAS 2008.

Noise correlations

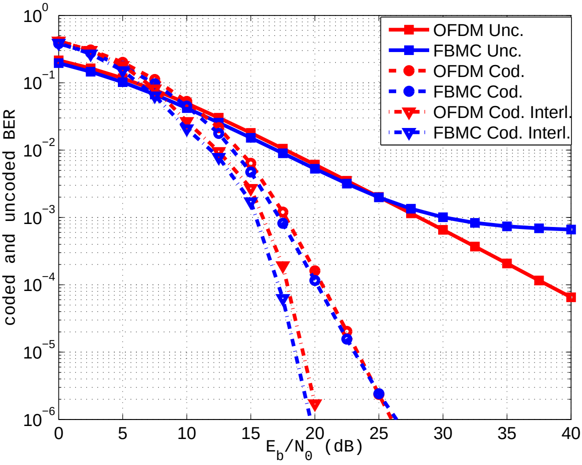
- Three distinguishable cases: Intra-subcarrier, Adjacent inter-subcarrier , Non-adjacent inter-subcarrier
- Ideal scenario (frequency flat channel and no equalizer)
  - Intra-subcarrier - Non existent
  - Adjacent inter-subcarrier - Non existent
  - Non-adjacent inter-subcarrier - Non existent
- Low frequency selective channel and one-tap equalizer
  - Intra-subcarrier - Non existent
  - Adjacent inter-subcarrier - small
  - Non-adjacent inter-subcarrier - very small (prototype dependent)
- High frequency selective channel and linear multitap equalizer
  - Intra-subcarrier - expected
  - Adjacent inter-subcarrier - expected
  - Non-adjacent inter-subcarrier - very small (prototype dependent)

Noise correlations

- Correlations may affect the coded BER performance
- A WiMAX simulation scenario was considered and the coded BER was evaluated
- Channel coding with and without interleaving was considered
- Convolutional encoder with native rate $1/2$, constraint length 7 and generator polynomials (IEEE Std. 802.16-2004): $g_1(D) = 1 + D + D^2 + D^3 + D^6$ and $g_2(D) = 1 + D^2 + D^3 + D^5 + D^6$
- Block interleaver of length $2^b \times M_{\text{used}}$ defined in IEEE Std. 802.16-2004
- A soft decoder (approximate LLR, “unquantized” inputs to the Viterbi decoder)

Coded BER Comparison between FBMC and CP-OFDM

Linear MMSE Eq. in WiMAX scenario



Parameters:

16-QAM

$C_\mathrm{r} = 1/2$

Conv. code

Soft decod.

$M = 1024$

$M_\mathrm{data} = 768$

$\Delta f = 10.9\,\mathrm{kHz}$

$BW = 10\,\mathrm{MHz}$

$T_\mathrm{s} = 89.28\,\mathrm{ns}$

$T_\mathrm{cp} = 91.42\,\mu\mathrm{s}(1/4)$

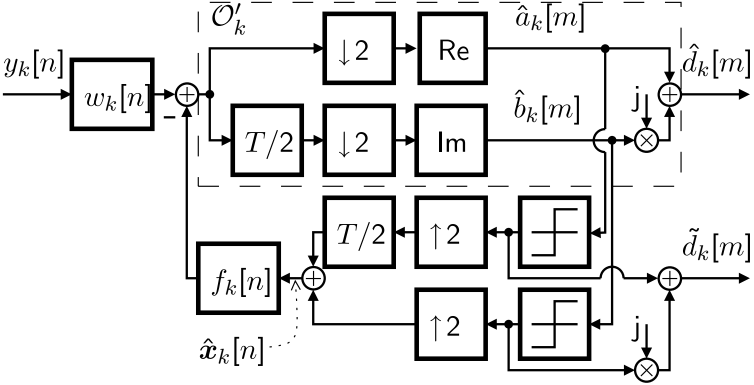
$N = 7$

ITU Veh. B Static

$\tau_\mathrm{RMS} = 4\,\mu\mathrm{s}$

MMSE Decision Feedback Equalizer for FBMC Systems

Per-subchannel DFE and OQAM de-staggering $\mathcal{O}'_k$, k even:



MMSE Decision Feedback Equalizer for FBMC Systems

Optimization problem:

$$(\boldsymbol{w}_{k,\mathsf{MMSE}}, \boldsymbol{f}_{k,\mathsf{MMSE}}) = \arg \min_{(\boldsymbol{w}_k, \boldsymbol{f}_k)} \mathsf{E}\left[|\hat{a}_k[m] - a_k[m-\nu]|^2 \right],$$

where

$$\hat{a}_k[m] = \mathsf{Re}\left[\boldsymbol{w}_k^{\mathsf{H}} \boldsymbol{y}_k[n] \right], \quad n = 2m, \quad m \in \mathbb{Z},$$

ν is the equalization delay and
$\boldsymbol{y}_k[n] = [y_k[n],\ y_k[n-1],\ \ldots,\ y_k[n-N+1]]^T \in \mathbb{C}^N$.

MMSE Decision Feedback Equalizer for FBMC Systems

DFE solution for i.i.d. input symbols $d_k[m]$ with variance σ_d^2:

$$\boxed{\begin{bmatrix} \boldsymbol{w}'_{k,\mathrm{MMSE}} \\ \boldsymbol{f}^{(\mathrm{R})}_{k,\mathrm{MMSE}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_k \boldsymbol{H}_k^{\mathsf{T}} + \boldsymbol{F}_k \boldsymbol{F}_k^{\mathsf{T}} + \boldsymbol{R}_{\eta,k} & -\frac{\sigma_d}{\sqrt{2}} \boldsymbol{H}_k \boldsymbol{J}_\nu \\ -\frac{\sigma_d}{\sqrt{2}} \boldsymbol{J}_\nu^T \boldsymbol{H}_k^T & \frac{\sigma_d^2}{2} \boldsymbol{I}_{B+1} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\sigma_d}{\sqrt{2}} \boldsymbol{H}_k \boldsymbol{e}_\nu \\ \boldsymbol{0}_{B+1} \end{bmatrix},}$$

where

$$\boldsymbol{J}_\nu = \begin{cases} \begin{bmatrix} \boldsymbol{0}_{(\nu+1)\times(B+1)} & \boldsymbol{I}_{B+1} & \boldsymbol{0}_{(L-B-\nu-1)\times(B+1)} \end{bmatrix}^{\mathsf{T}}, & L-\nu > B+1, \\ \begin{bmatrix} \boldsymbol{0}_{(\nu+1)\times(B+1)} & \begin{matrix} \boldsymbol{I}_{B+1} \\ \boldsymbol{0}_{(L-\nu)\times(B+\nu-L+1)} \end{matrix} \end{bmatrix}^{\mathsf{T}}, & L-\nu < B+1, \\ \begin{bmatrix} \boldsymbol{0}_{(\nu+1)\times(B+1)} & \boldsymbol{I}_{B+1} \end{bmatrix}^{\mathsf{T}}, & L-\nu = B+1. \end{cases}$$

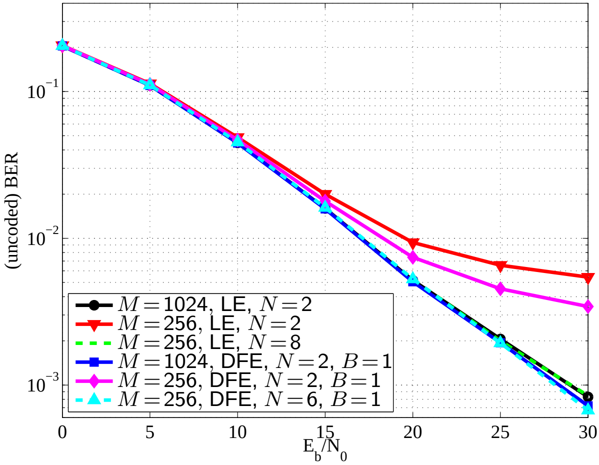
B: length of the feedback filter

$L = N + Q - 2$

Baltar, L.G.; Waldhauser, D.S.; Nossek, J.A.; *MMSE subchannel decision feedback equalization for filter bank based multicarrier systems*; ISCAS 2009.

BER Comparison between FBMC and CP-OFDM

MMSE DFE in WiMAX scenario:



Parameters:

16-QAM

$M = 1024(256)$

$M_{\text{data}} = 840(210)$

$\Delta f = 10.9(43.7)\,\text{kHz}$

$BW = 10\,\text{MHz}$

$T_{\text{s}} = 89.28\,\text{ns}$

$T_{\text{cp}} = 91.42\,\mu\text{s}(1/4)$

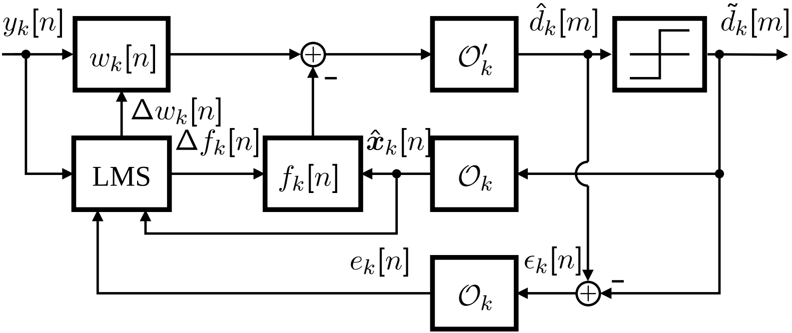
Extended TU 3GPP

Static

$\tau_{\text{RMS}} = 0.99\,\mu\text{s}$

Adaptive MMSE DF Equalizer for FBMC Systems

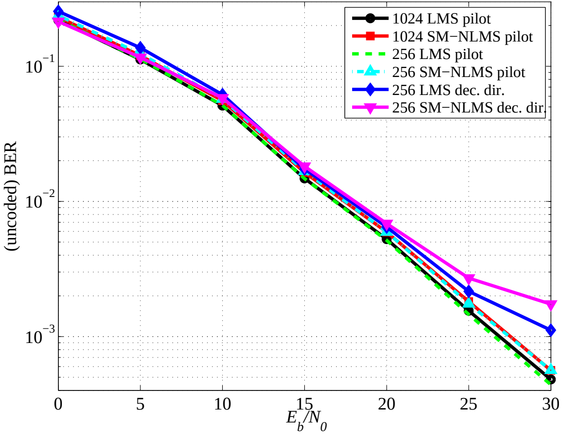
Per-subchannel decision directed OQAM adaptive DFE:



Waldhauser, D.S.; Baltar, L.G.; Nossek, J.A.; *Adaptive decision feedback equalization for filter bank based multicarrier systems*; ISCAS 2009.

Adaptive DF Equalizer for FBMC Systems

Adaptive O-QAM DFE LMS in a static WiMAX scenario:



Parameters:

16-QAM

$M = 1024(256)$

$M_{\text{used}} = 840(210)$

$\Delta f = 10.9(43.7)\,\text{kHz}$

$B = 10\,\text{MHz}$

$T_{\text{s}} = 89.28\,\text{ns}$

$T_{\text{cp}} = 91.42\,\mu\text{s}(1/4)$

ITU Veh. A Static

$\tau_{\text{RMS}} = 0.37\,\mu\text{s}$

Conclusions

- The effect of noise correlations in a coded FBMC system is negligible for a WiMAX scenario
- DFE should be employed only when the frequency selectivity inside one subcarrier is very high
- The equalizer analytical solutions are based on perfect channel knowledge